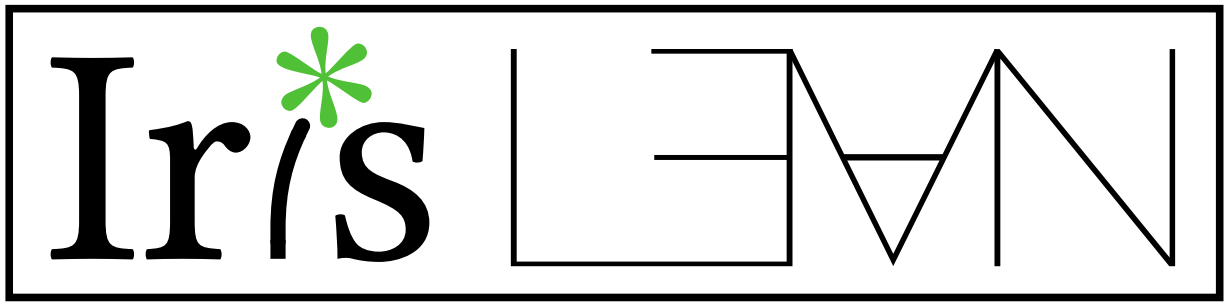
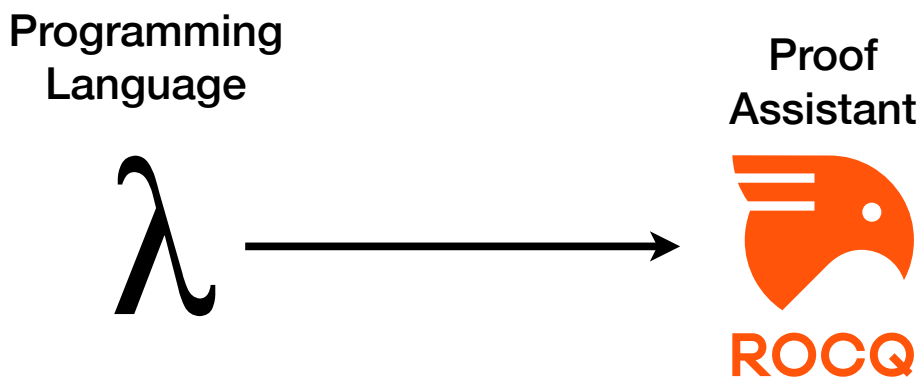


Markus de Medeiros

NYU

NJPLS, December 5th 2025


Programming
Language
λ
Proof
Assistant
ROCQ

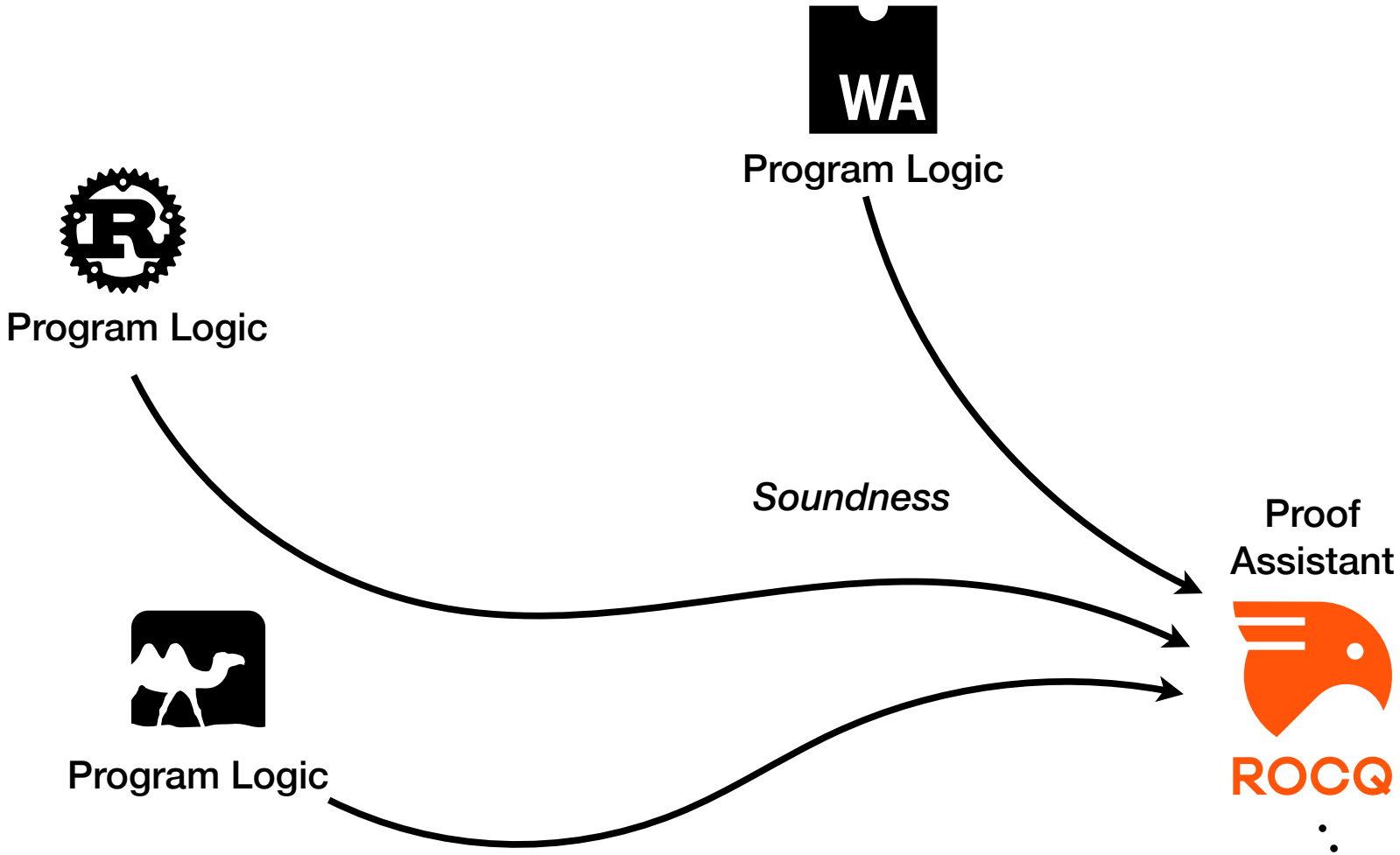
Program Logic

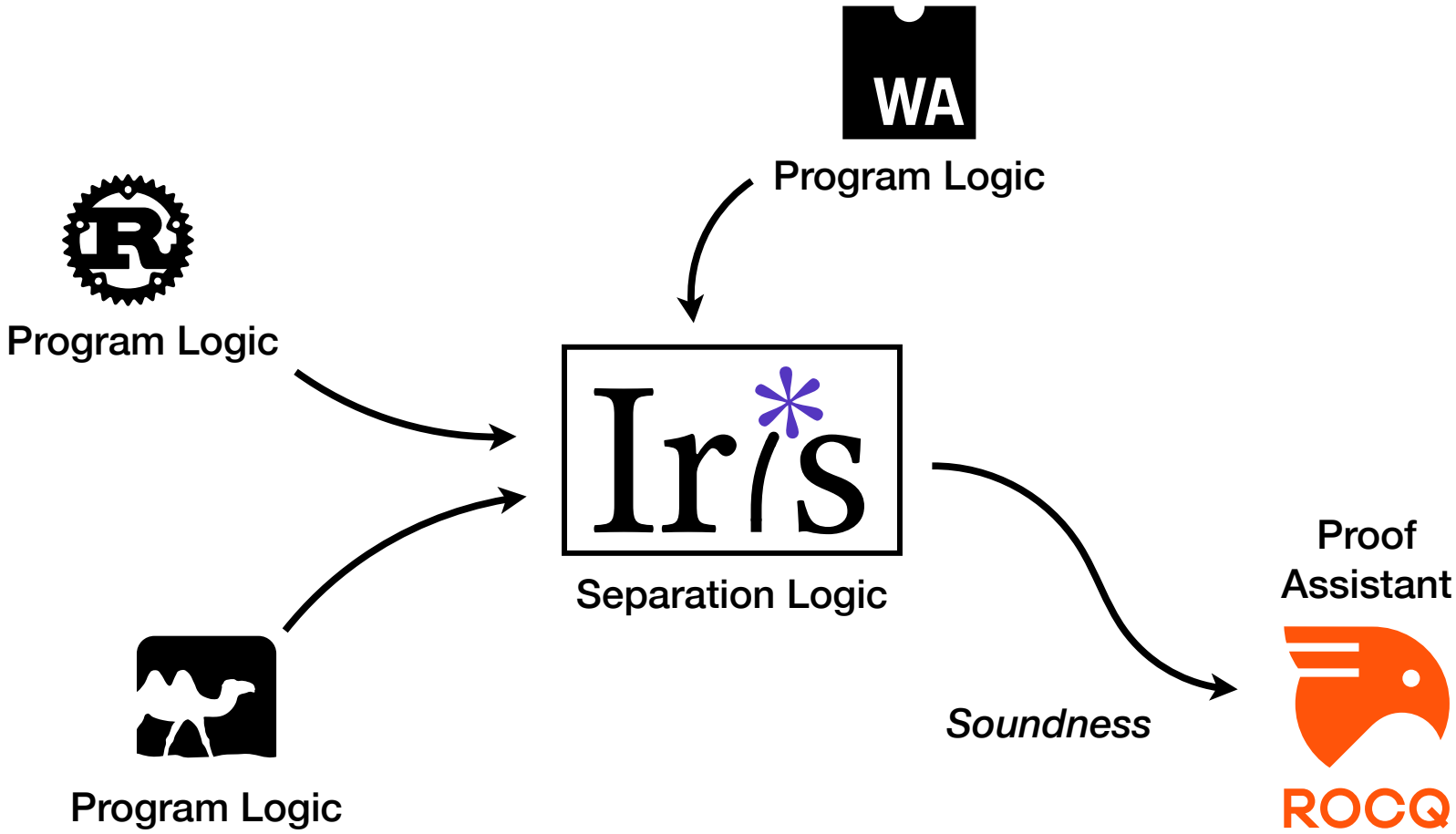
Program Logic

Program Logic

Proof
Assistant

ROCQ


WA
Program Logic
Program Logic
Soundness
Proof
Assistant
Program Logic
ROCQ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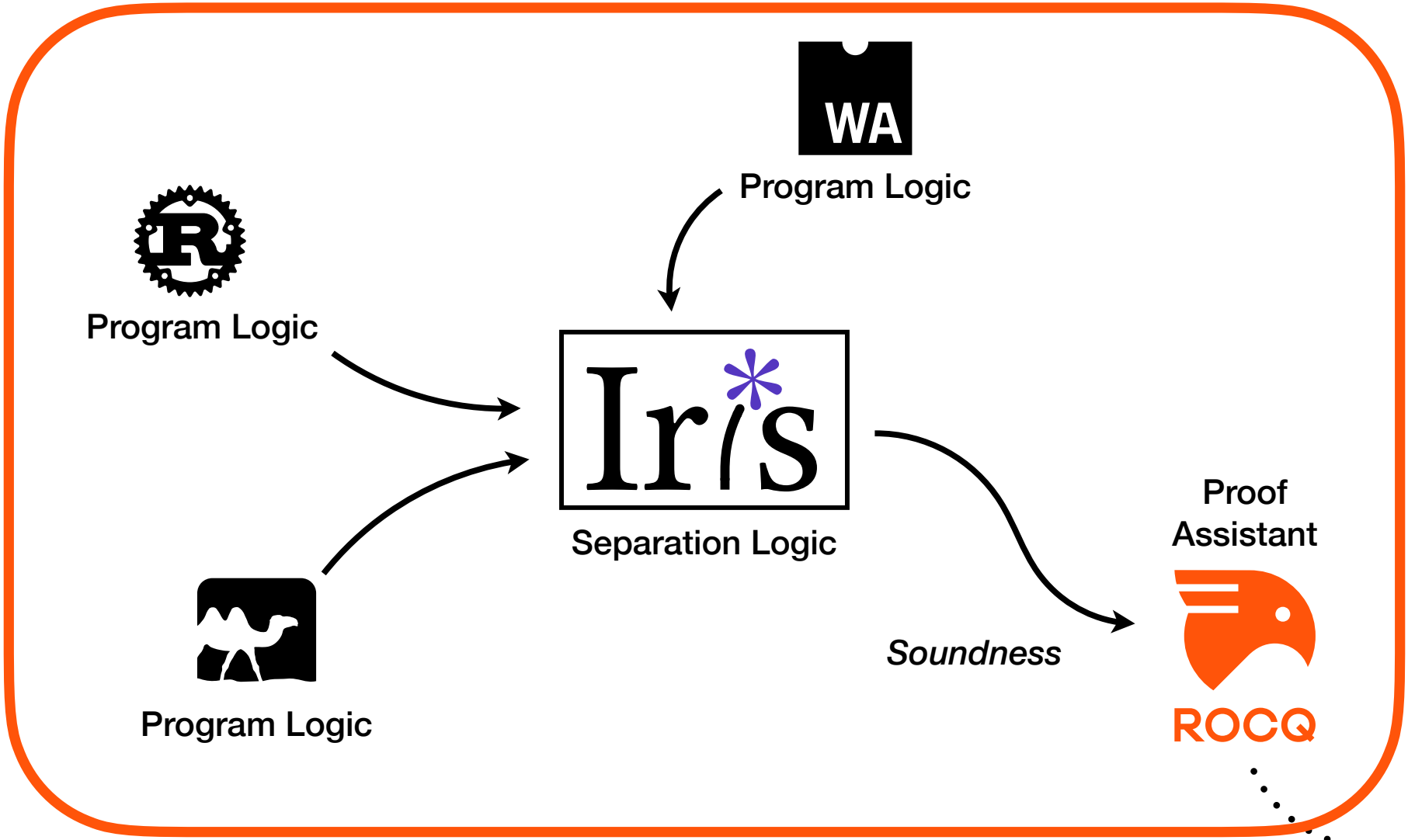
WA
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Program Logic
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Separation Logic
Program Logic
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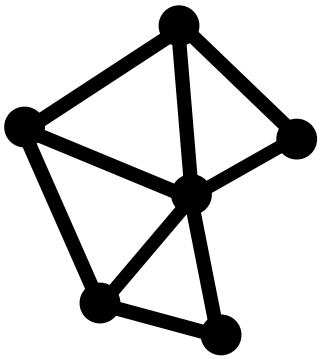
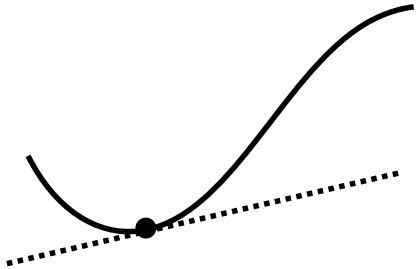
Graph Theory

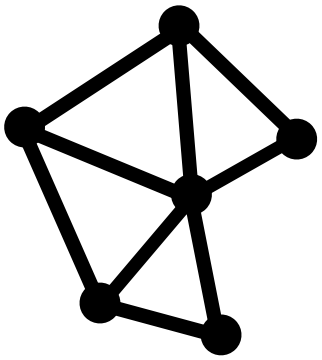


Calculus

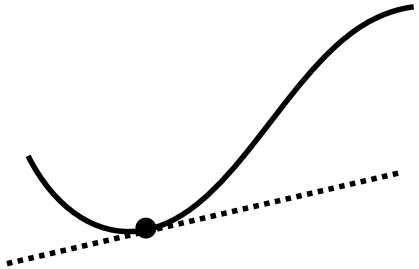


ROCQ

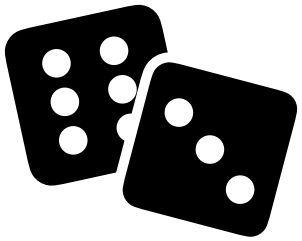
Graph Theory



Calculus



Probability



ROCQ

Formalized Math

	Coquelicot*	2,000	Theorems	40,000	LOC
	Mathcomp-analysis + Mathcomp*	22,000	Theorems	182,000	LOC
	Mathlib[1]	243,000	Theorems	2,000,000	LOC
	Archive of Formal Proofs[2]	300,000	Theorems	2,500,000	LOC
	Iris*	4,800	Theorems	53,000	LOC

1 https://www.isa-afp.org/statistics/
2 https://leanprover-community.github.io/mathlib_stats.html
** Unofficial Estimate (grep)*

Many other projects not included!

Formalized Math

ROCQ	Coquelicot*	2,000	Theorems	40,000	LOC
ROCQ	Mathcomp-analysis + Mathcomp*	22,000	Theorems	182,000	LOC
LEAN THEOREM PROVER	Mathlib[1]	243,000	Theorems	2,000,000	LOC
Isabelle	Archive of Formal Proofs[2]	300,000	Theorems	2,500,000	LOC
Iris	Iris*	4,800	Theorems	53,000	LOC

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Formalized Math

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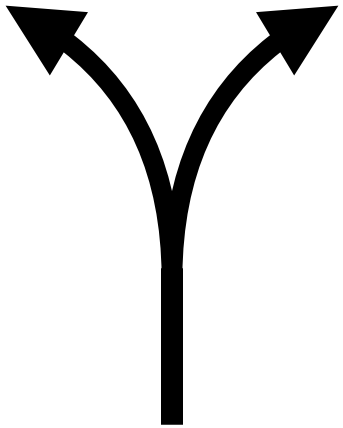
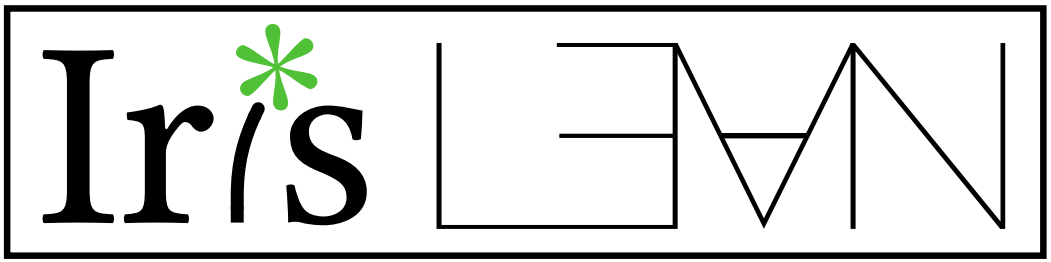
Many other projects not included!

A Crossroads

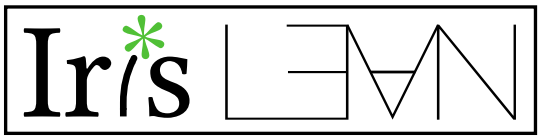
Math in ROCQ

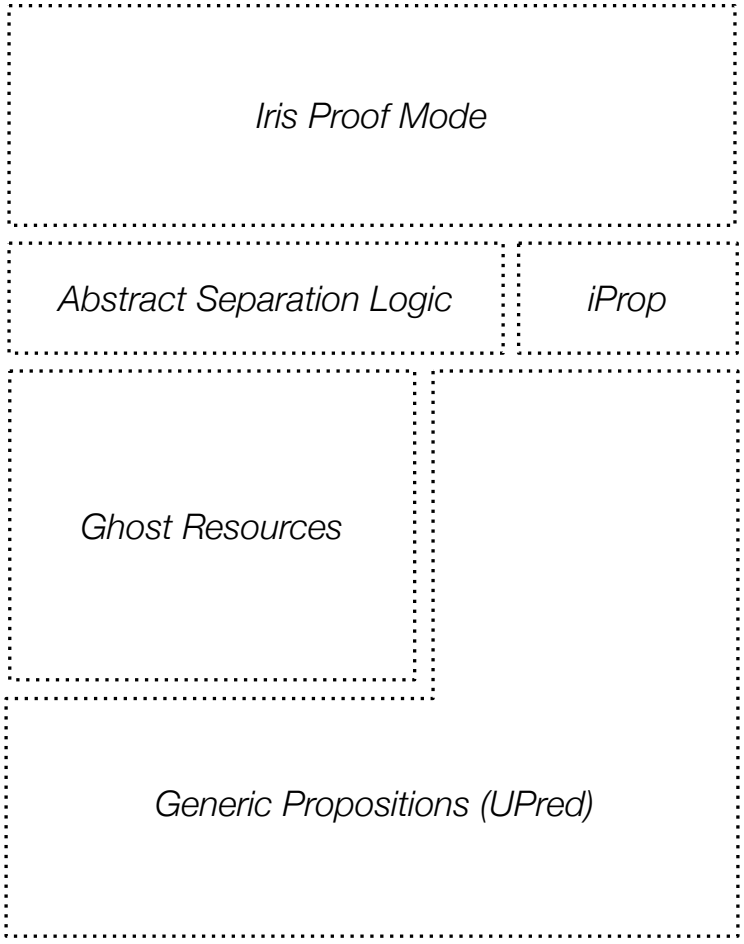


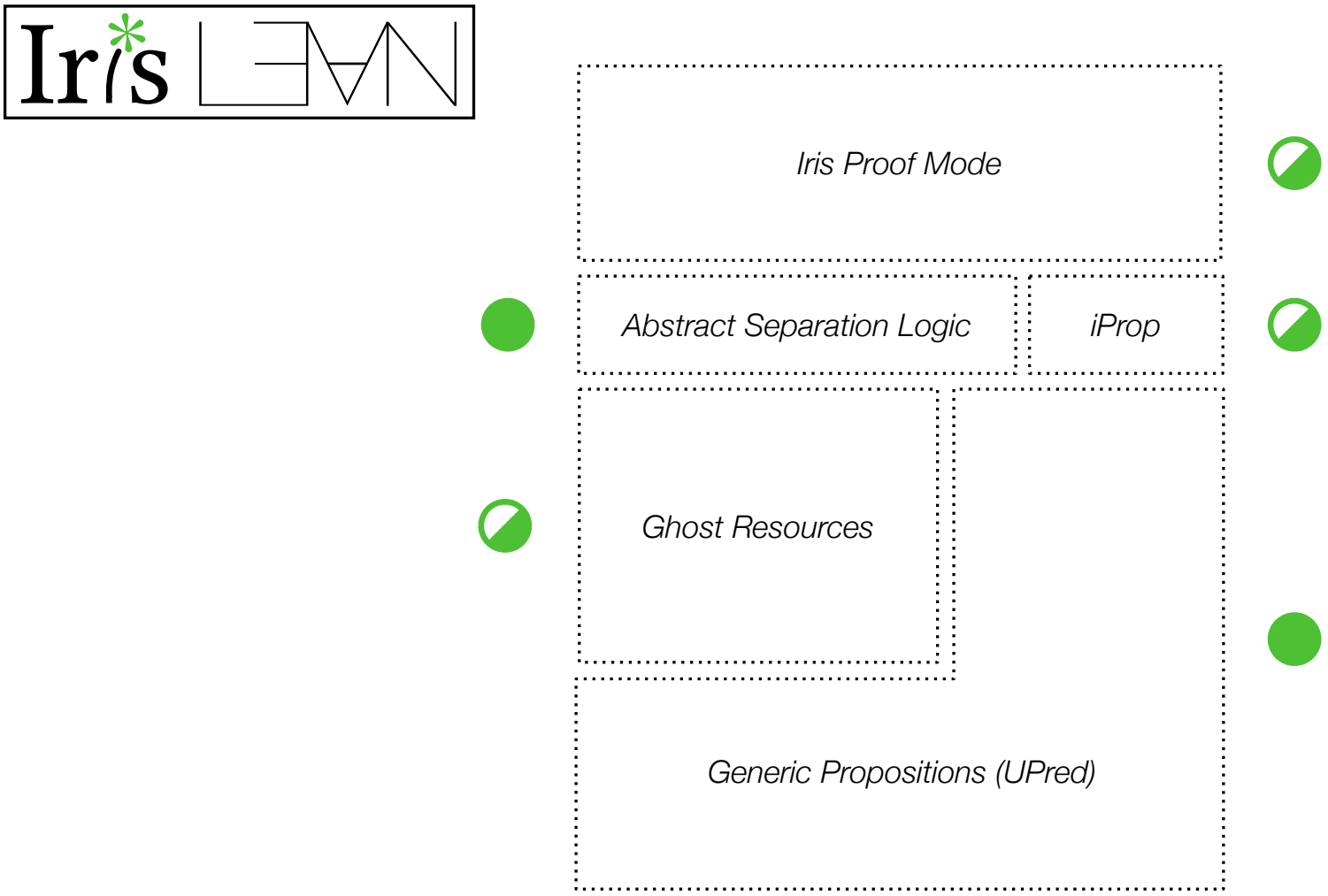
Iris in ?



- Possible in Lean?
- Iris-Lean adaptations?


Iris


Iris Proof Mode
Abstract Separation Logic
iProp
Ghost Resources
Generic Propositions (UPred)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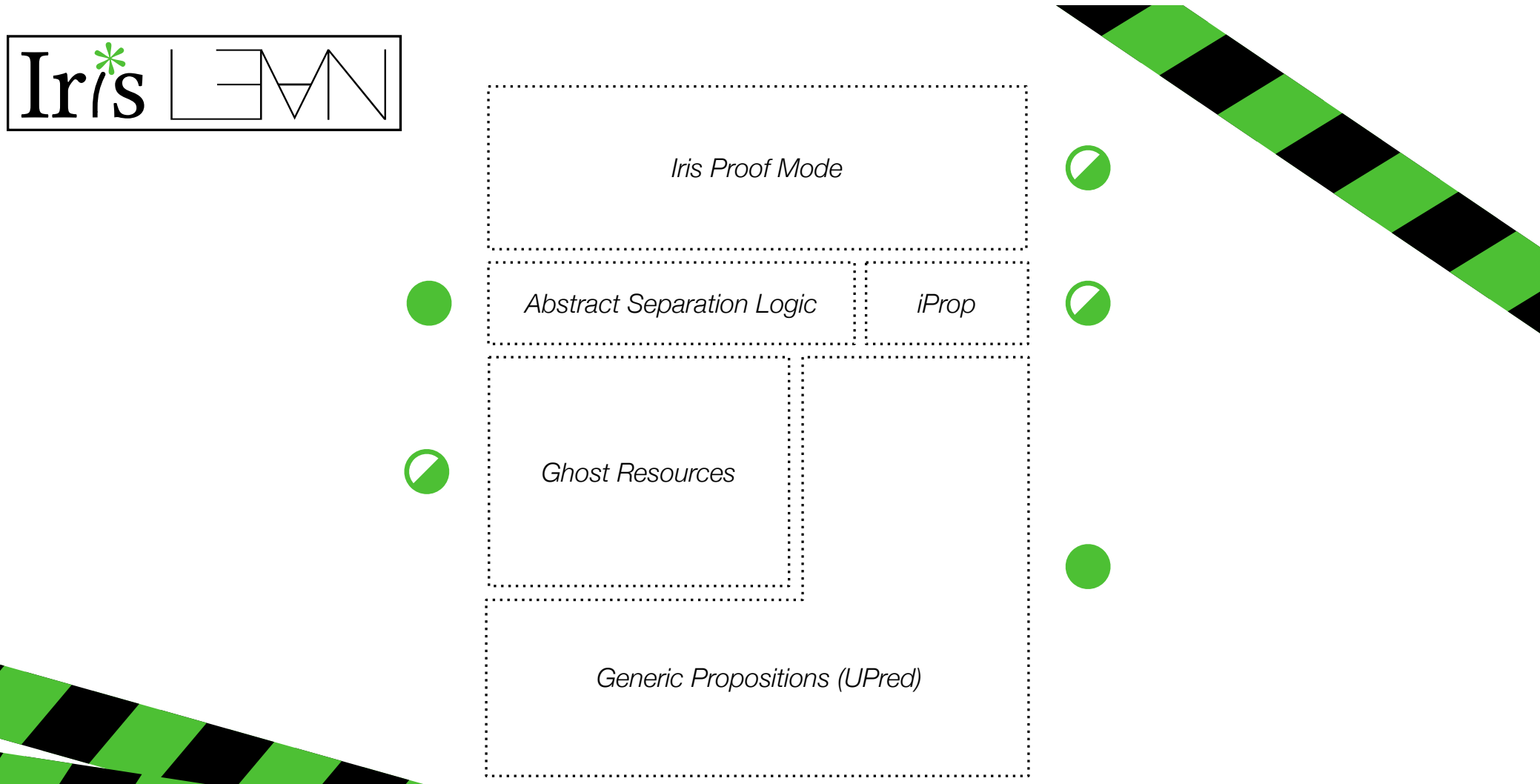
Iris Proof Mode
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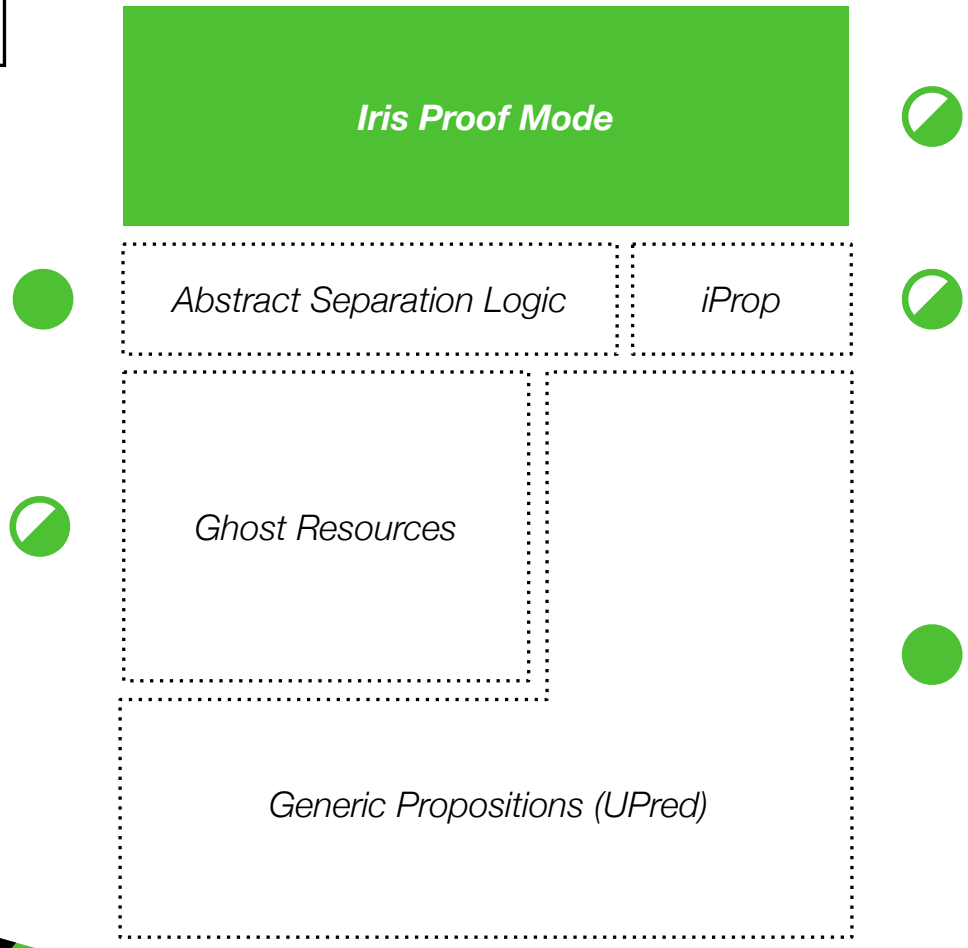
Iris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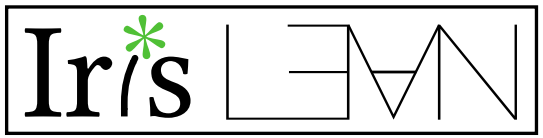


Iris Proof Mode

Karlsruher Institut für Technologie

Institut für Programmstrukturen
und Datenorganisation (IPD)

Lehrstuhl Prof. Dr.-Ing. Snelting

An Improved Interface for Interactive Proofs in Separation Logic

Masterarbeit von

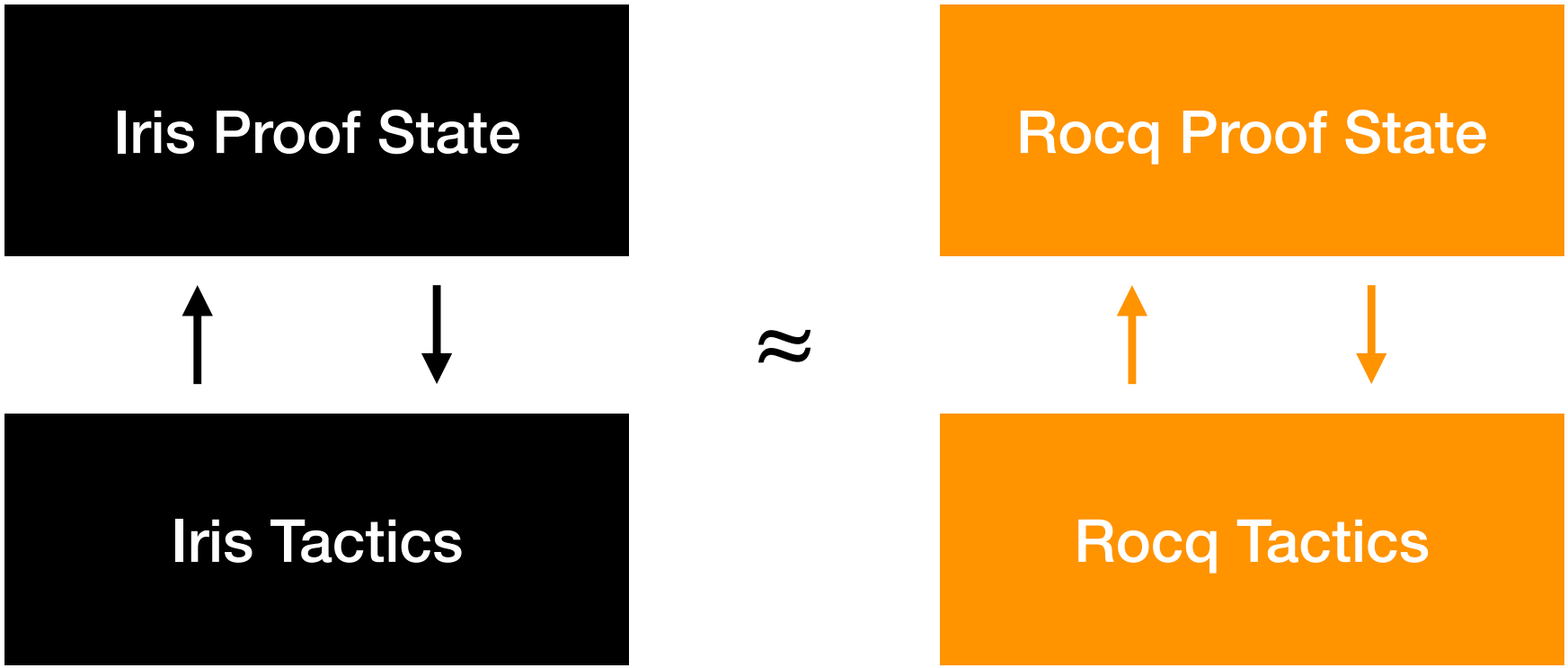
Lars König

an der Fakultät für Informatik

```
theorem proof_example [BI PROP] (P Q R : PROP) (Φ : α → PROP) :
    P ∗ Q ∗ □ R ⊢ □ (R -∗ ∃ x, Φ x) -∗ ∃ x, Φ x ∗ P ∗ Q
:= by                                   Iris Proof Mode
    iintro ⟨HP, HQ, #HR⟩ #HRΦ           -------------------------
    ispecialize HRΦ HR as HΦ            HR : R
    icases HΦ with ⟨x, HΦ⟩              HRΦ : R -∗ ∃ x, Φ x
    iexists x                           HΦ : ∃ x, Φ x
    isplit r                            ------------------------- □
    · iassumption                       HP : P
    isplit l [HP]                       HQ : Q
    · iexact HP                         ------------------------- ∗
    · iexact HQ                         ∃ x, Φ x ∗ P ∗ Q
```

Erstgutachter:	Prof. Dr.-Ing. Gregor Snelting
Zweitgutachter:	Prof. Dr. rer. nat. Bernhard Beckert
Betreuender Mitarbeiter:	M. Sc. Sebastian Ullrich
Abgabedatum:	30. September 2022

Iris Proof Mode



Iris Proof Mode

```
Theorem sep_mp {P Q : iProp Σ} :
  ⊢ P ∗ (P -∗ Q) -∗ Q.
Proof.
  iIntros "[HP Hwand]".
  iApply "Hwand".
  iExact "HP".
Qed.
```

```
P, Q : iProp Σ
————————————————————————————————
P ∗ (P -∗ Q) -∗ Q
```

Iris Proof Mode

```
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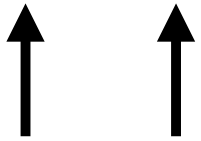

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Qed.
```

```
P, Q : iProp Σ
──────────────────────────────────
  "HP" : P
  "Hwand" : P -∗ Q
  ──────────────────────────────∗
  Q
```

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```
Theorem sep_mp {P Q : iProp Σ} :
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Proof.
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  iExact "HP".
Qed.
```

```
P, Q : iProp Σ
──────────────────────────────
  "HP" : P
 ──────────────────────────∗
  P
```

Iris Proof Mode

```
Theorem sep_mp {P Q : iProp Σ} :
  ⊢ P ∗ (P -∗ Q) -∗ Q.
Proof.
  iIntros "[HP Hwand]".
  iApply "Hwand".
  iExact "HP".
Qed.
```

Iris Proof Mode

Iris Proof State

```
"HP" : P
"Hwand" : P -* Q
————————————————————*
Q
```

Iris Tactics

```
iIntros "[HP Hwand]".
iApply "Hwand".
iExact "HP".
```

Iris Proof Mode

Iris Proof State

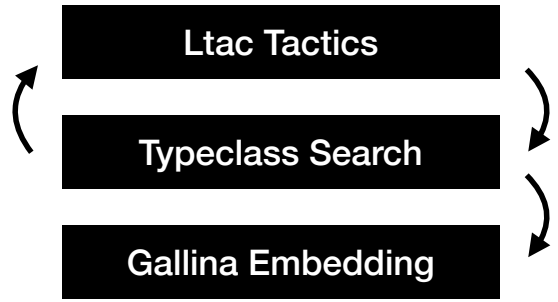
```
"HP" : P
"Hwand" : P -* Q
————————————————————*
Q
```

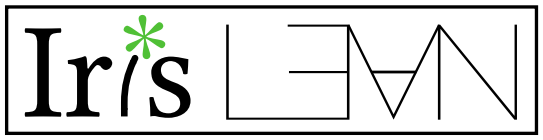
Iris Tactics

```
iIntros "[HP Hwand]".
iApply "Hwand".
iExact "HP".
```

Rocq Complexity

```
envs_entails {|
  env_intuitionistic := Enil;
  env_spatial := Esnoc (Esnoc Enil
    "HP" P)
    "Hwand" (bi_wand P Q);
   env_counter := 2%positive |}
Q
```





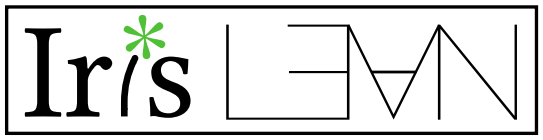
Iris Proof Mode

Iris Proof State

```
*HP : P
*Hwand : P -* Q
  ⊢ Q
```

Iris Tactics

```
iintros ⟨HP, Hwand⟩
iapply Hwand
iexact HP
```



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```
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  ⊢ Q
```
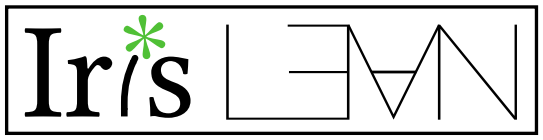

Iris Tactics

```
iintros ⟨HP, Hwand⟩
iapply Hwand
iexact HP
```

```
⊢ Entails (sep P (wand P Q))) Q
```

\+ Simpler encoding



Iris Proof Mode

Iris Proof State

```
*HP : P
*Hwand : P -* Q
  ⊢ Q
```

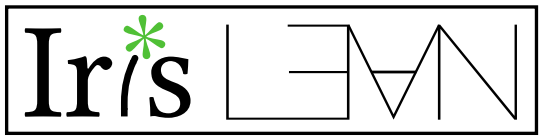
```
⊢ Entails (sep P (wand P Q))) Q
```

+ Simpler encoding

Iris Tactics

```
iintros ⟨HP, Hwand⟩
iapply Hwand
iexact HP
```

+ Monadic, imperative metaprograms



Iris Proof Mode

Iris Proof State

```
*HP : P
*Hwand : P -* Q
  ⊢ Q
```

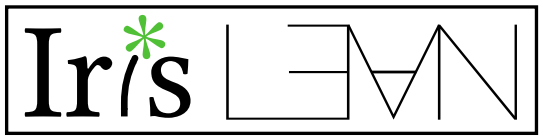
Iris Tactics

```
iintros ⟨HP, Hwand⟩
iapply Hwand
iexact HP
```

```
⊢ Entails (sep P (wand P Q))) Q
```

- \+ Simpler encoding

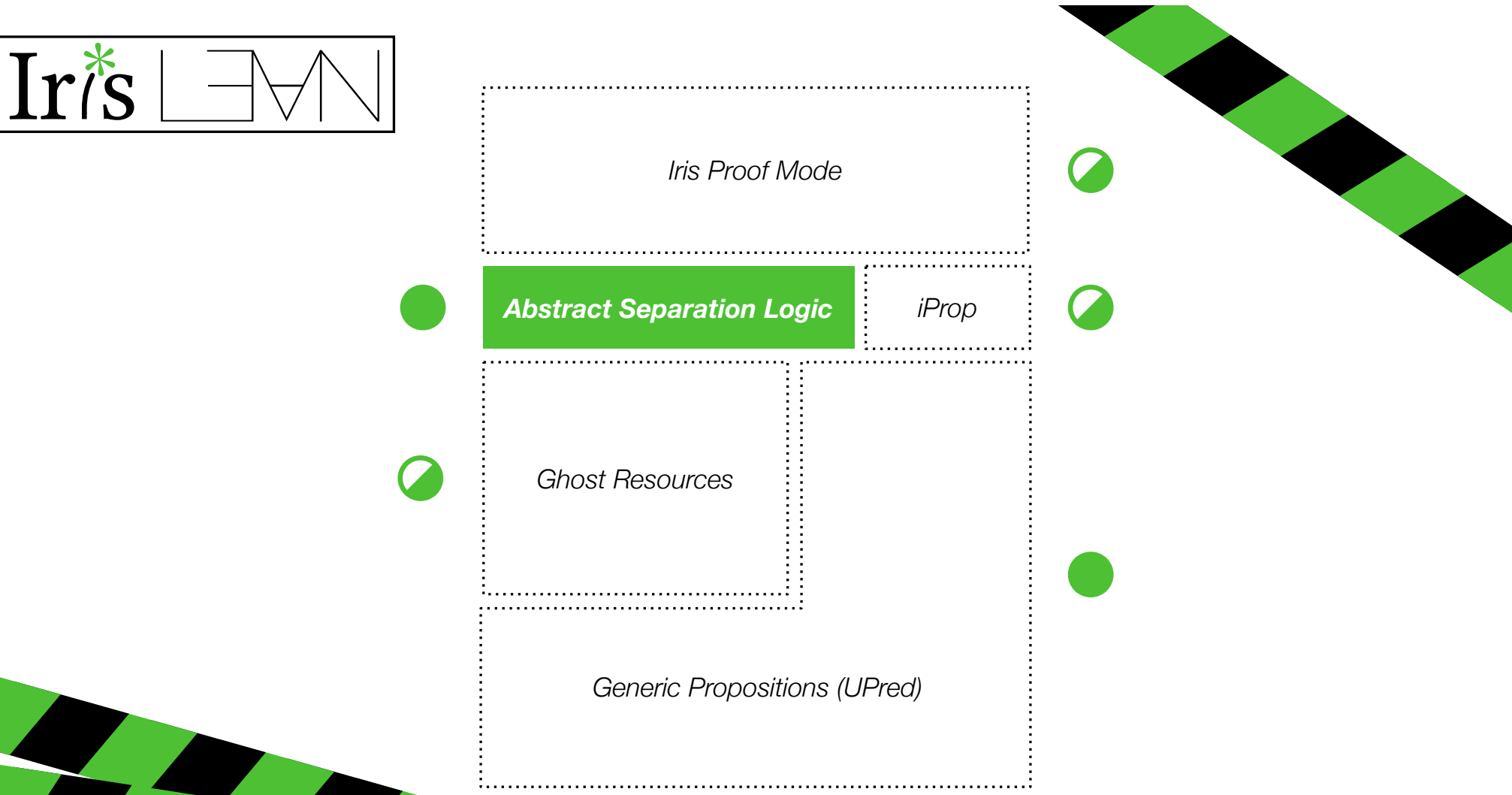
- \+ Monadic, imperative metaprograms
- ? Typeclasses?
- ? Faster?



Iris Proof Mode

- Intro Patterns
- Cases Patterns
- Selection Patterns

- `iintro`
- `iexists`
- `ileft/iright`
- `ipure_intro`
- `iex_falso`
- `iexact`
- `ispecialize`
- `isplit`
- `icases`
- `iapply`
- `imod`
- `iinduction`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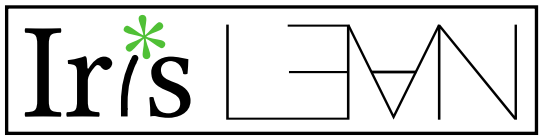
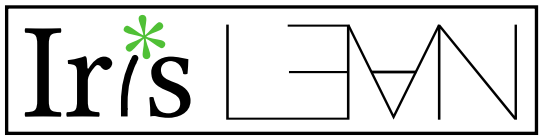


Abstract Separation Logic

- Interface for the syntax of a separation logic

Terms of the logic

```
class BIBase (PROP : Type u) where
  Entails : PROP → PROP → Prop
  emp : PROP
  pure : Prop → PROP
  and : PROP → PROP → PROP
  or : PROP → PROP → PROP
  imp : PROP → PROP → PROP
  …
```

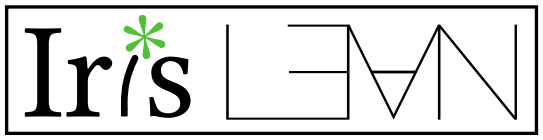


Abstract Separation Logic

“There exists (a : A) such that f(a) holds”

```
exists {A} : (f : A → PROP) → PROP
```

Function



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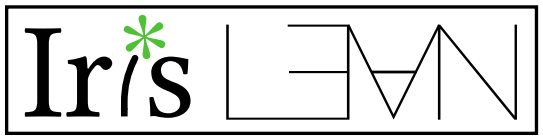


Abstract Separation Logic

“There exists (a : A) such that f(a) holds”

```
exists {A} : (f : A → PROP) → PROP
```

Universe Levels

“There exists some (a ∈ S) which holds”

```
sExists : (S : Set PROP) → PROP
```

Iris Lean

Abstract Separation Logic

"There exists (a : A) such that f(a) holds"

```
exists {A} : (f : A → PROP) → PROP
```

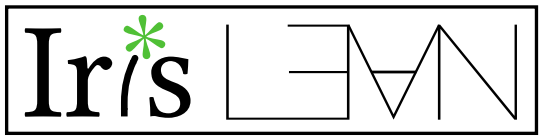
Universe Levels

"There exists some (a ∈ S) which holds"

```
sExists : (S : Set PROP) → PROP
```

```
exists (f : A → PROP) :=
  sExists (range f)
```

All Universes OK



“There exists (a : A) such that f(a) holds”

```
exists {A} : (f : A → PROP) → PROP
```

Universe Levels

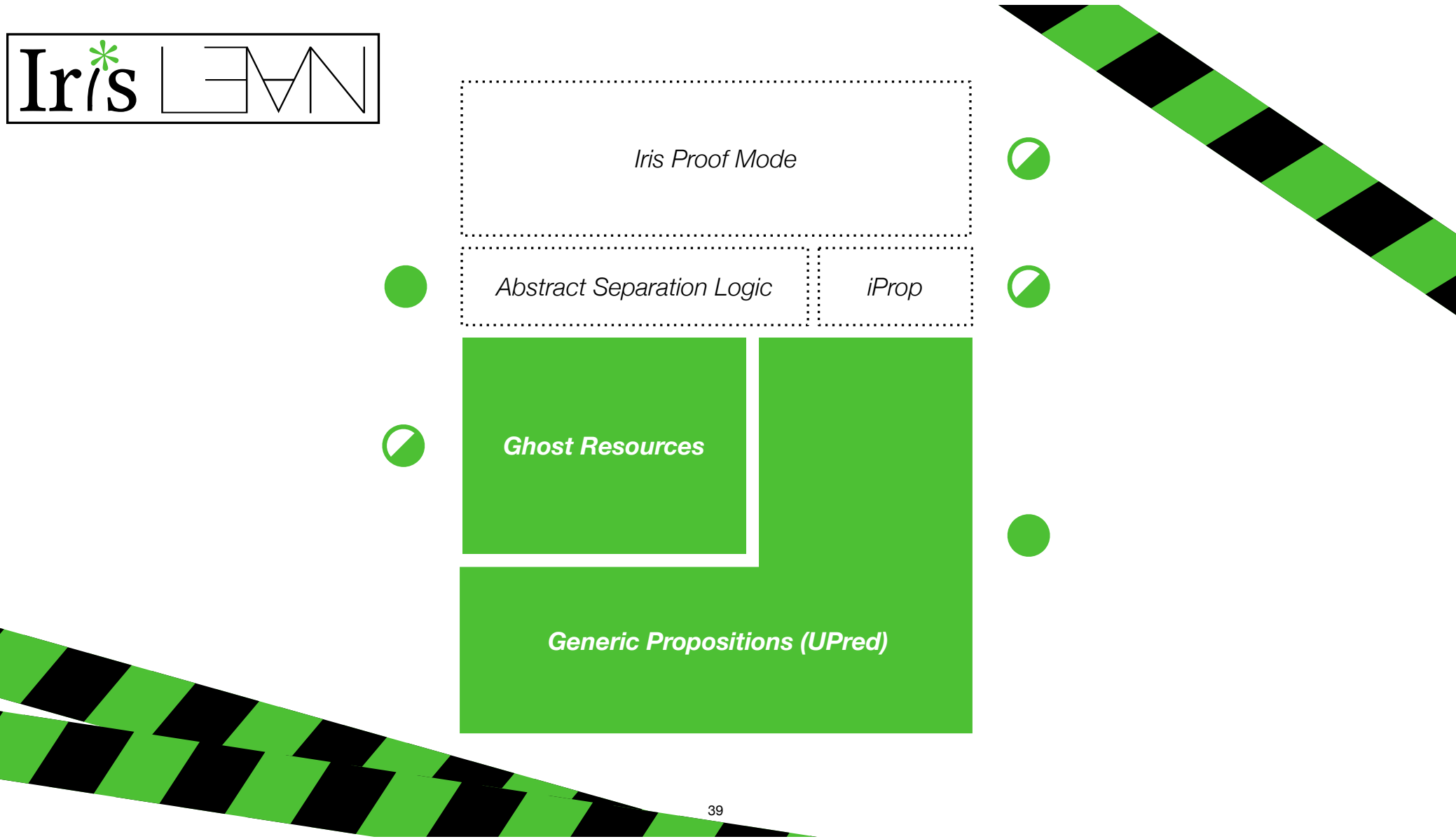
```
exists (f : A → PROP) :=
  sExists (range f)
```

All Universes OK

“There exists some (a ∈ S) which holds”

```
sExists : (S : Set PROP) → PROP
```

- ? Does everything generalize?
- ? Set quantifiers in Iris-Rocq?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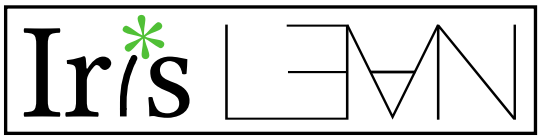
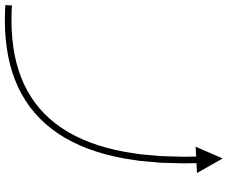
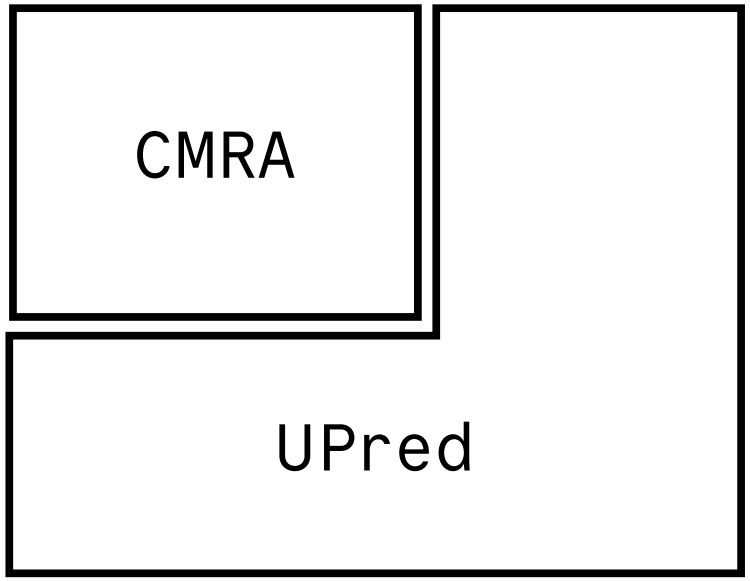


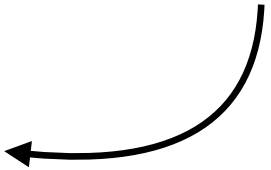
Base Logic

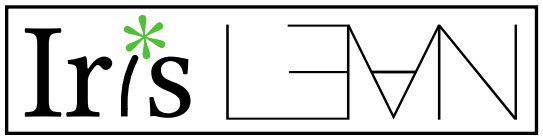
Algebraic abstraction





Step-indexed, higher order, separation logic





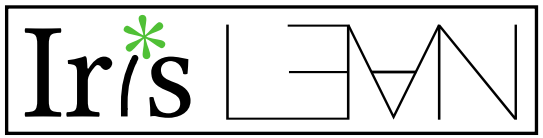
Base Logic

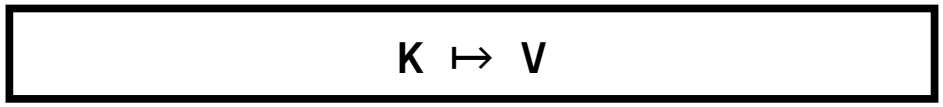
A library of CMRA combinators

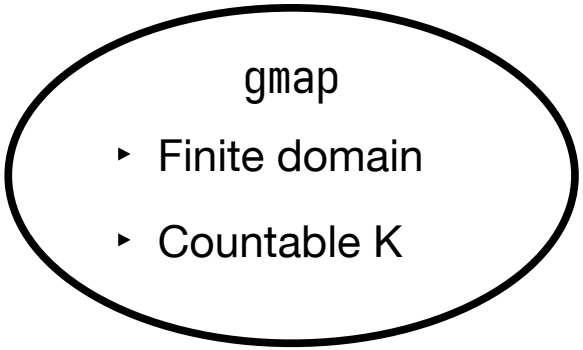
- `auth`
- `csum`
- `gset`
- `reservation_map`
- `vectors`

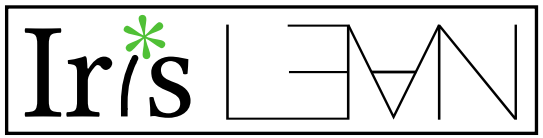
… & many more

- `excl`
- `agree`
- `frac` ***Generalized!***
- `dfrac`
- `view`
- `gen_map` ***New!***


Iris
Ghost Resources

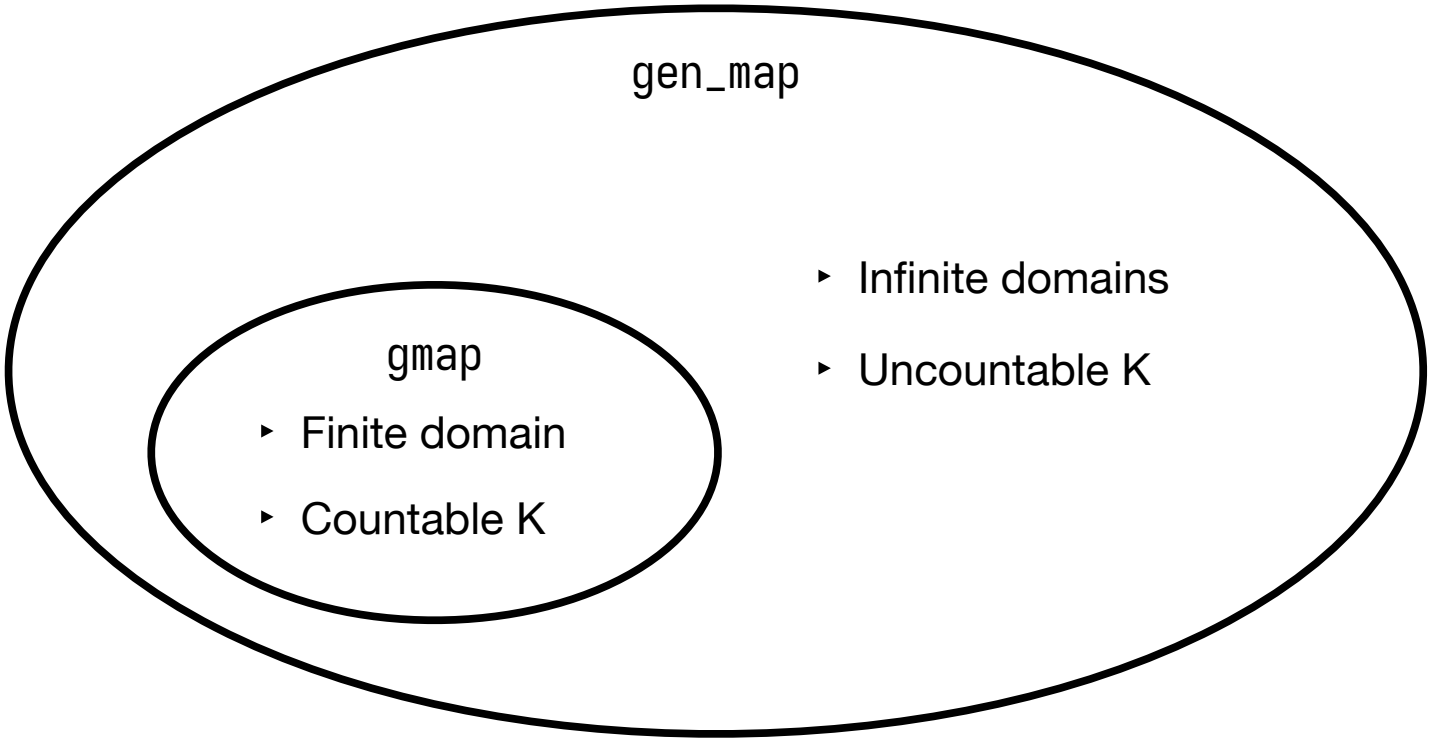

K ↦ V


gmap
Finite domain
Countable K



Ghost Resources

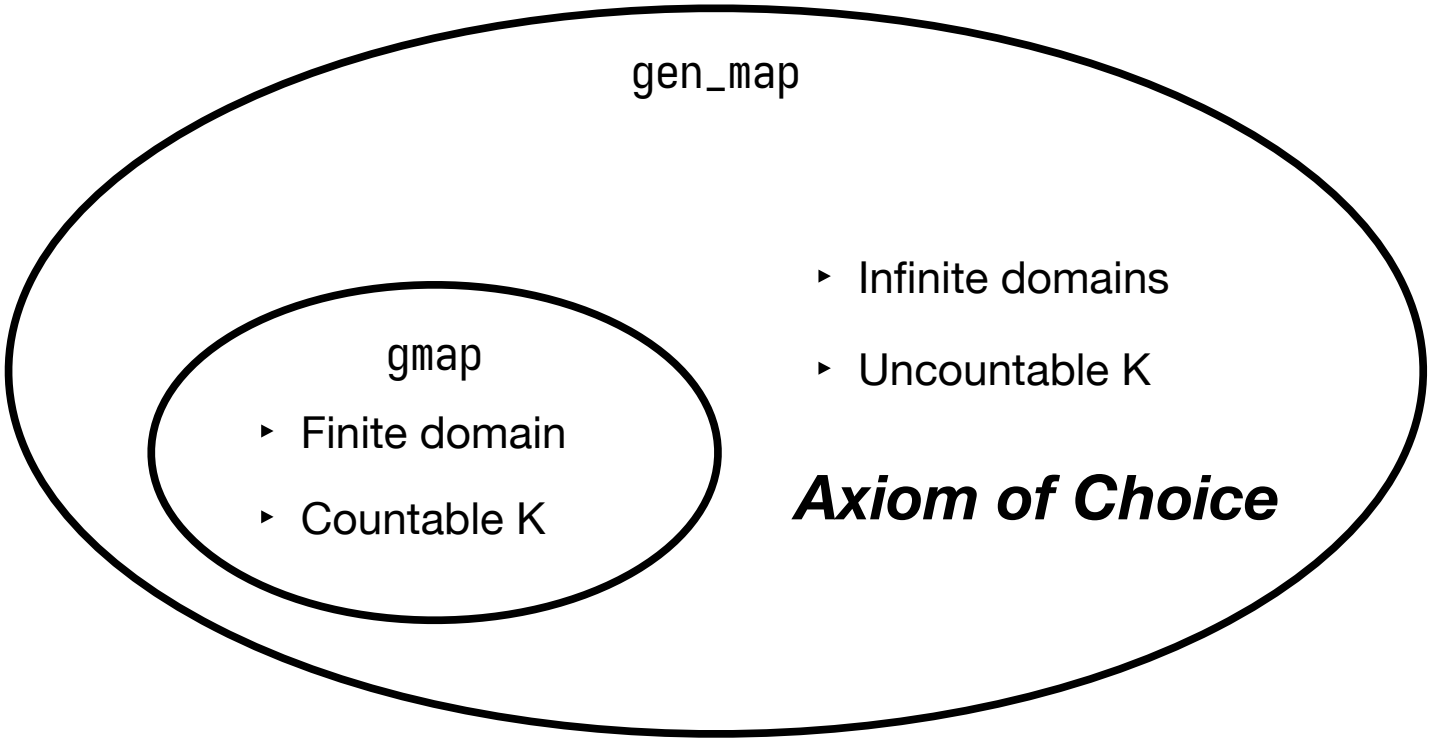
$K \mapsto V$

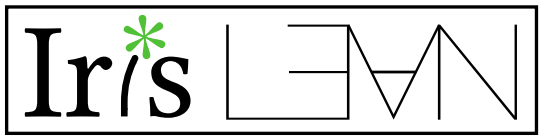


Iris

Ghost Resources

$K \mapsto V$

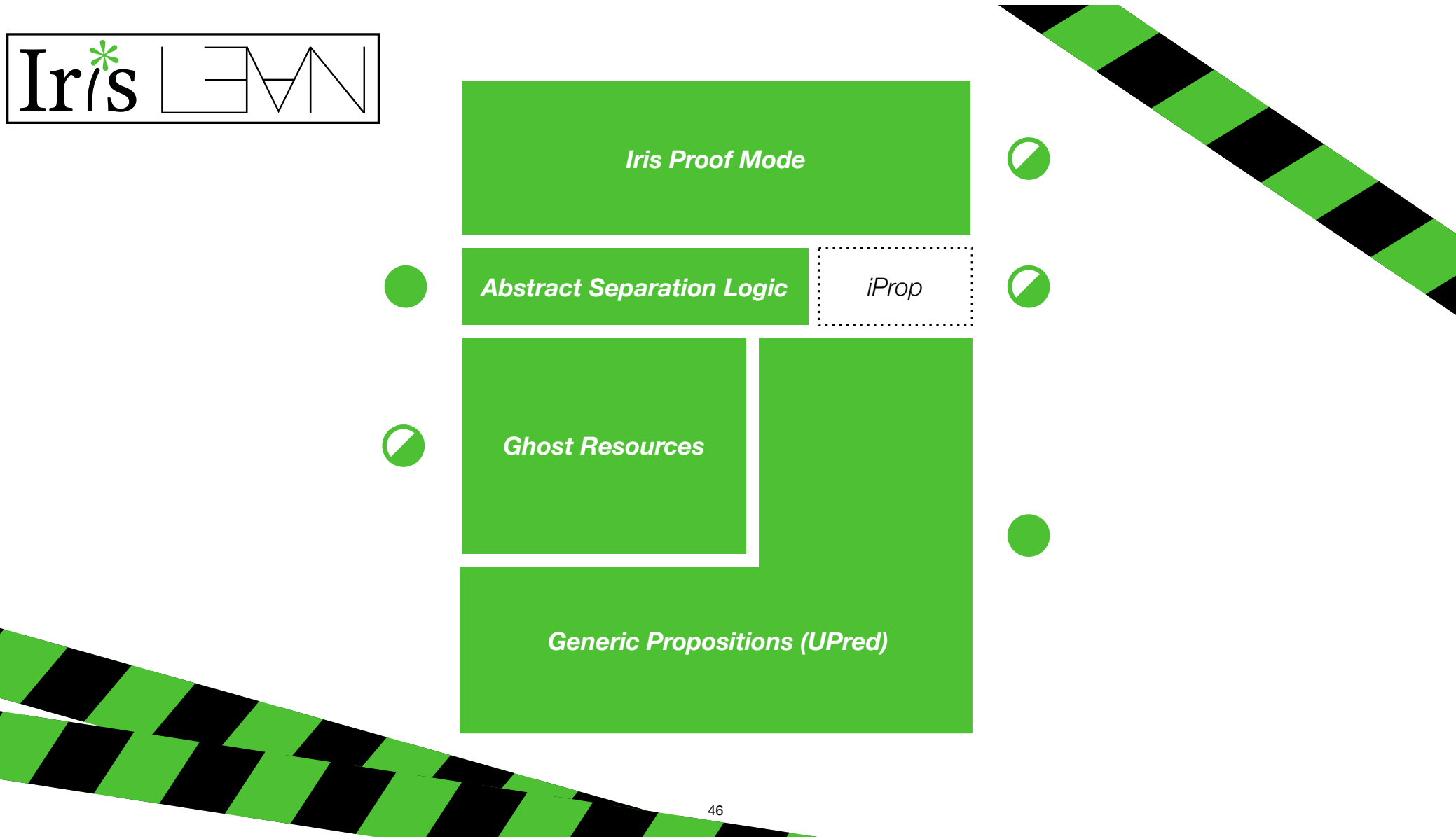


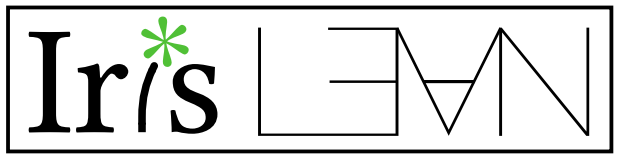


Ghost Resources

Mathematical Resources

```
variable {α : Type _} [MeasurableSpace α]
instance : CMRA (Measure α) where
 op μ₁ μ₂ := μ₁ + μ₂
  Valid μ := μ .univ ≤ 1
  validN_op_left :=
     (le_of_add_le_of_nonneg_left · <| zero_le _)
  assoc := by simp [add_assoc]
  comm := by simp [add_comm]
```


Iris Proof Mode
Abstract Separation Logic
iProp
Ghost Resources
Generic Propositions (UPred)



Works in Progress

Bluebell[1] Port

- Ethereum Foundation
- Iris + Mathlib Probability
- Independence CMRA

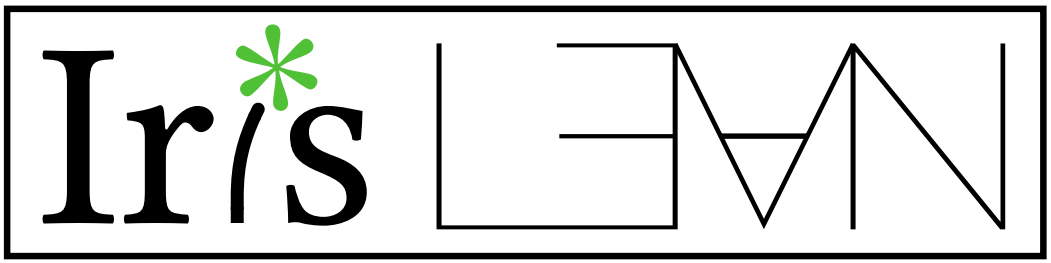
Nola[2] Port

- Microsoft Research
- With Aeneas (Rust)

Neuron Relational Logic

- Amazon Web Services
- Simple relational logic
- Full soundness proof

1 Jialu, D'Osualdo, Azadeh. Bluebell: An Alliance of Relational Lifting and Independence for Probabilistic Reasoning
2 Matsushita, Tsukada. Nola: Later-Free Ghost State for Verifying Termination in Iris



- Possible in Lean?
  - Yes!
- Iris-Lean adaptations?
  - Simplified Proof Mode
  - Set-Based Quantifiers
  - Generalized Maps

Contributors

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- *Alex Bai*
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- *Joe Watt*
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- *Lars König*
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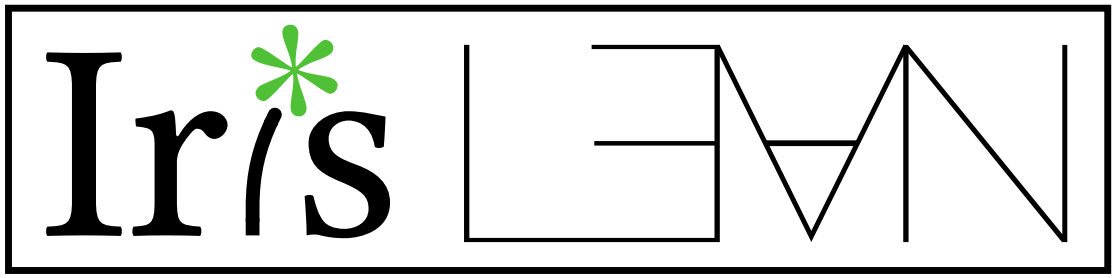
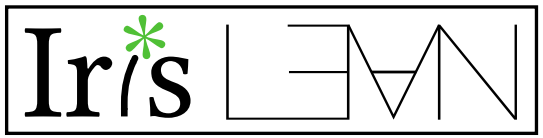
- *Markus de Medeiros*



Github: `leanprover-community/iris-lean`

Lean Zulip: `#iris-lean`

Email: `mjd9606@nyu.edu`



iProp

```
abbrev F0 : OFunctorPre := constOF (Agree (LeibnizO String))
variable {GF} [E0 : ElemG GF F0]

example : ⊢ |==> ∃ (γ0 γ1 : GName) (s0 s1 : String),
    iOwn (E := E0) γ0 (toAgree ⟨s0⟩) ∗
    iOwn (E := E0) γ1 (toAgree ⟨s1⟩) := by
  let v0 : F0.ap (IProp GF) := toAgree ⟨"string0"⟩
  let v1 : F0.ap (IProp GF) := toAgree ⟨"string1"⟩
  refine emp_sep.mpr.trans <| (sep_mono (iOwn_alloc v1 (fun _ => trivial)) .rfl).trans ?_
  refine emp_sep.mpr.trans <| (sep_mono (iOwn_alloc v0 (fun _ => trivial)) .rfl).trans ?_
  -- Eliminate the bupds (by hand, until iMod is implemented)
  -- …
  istart
  iintro ⟨⟨γ0, Hγ0⟩, ⟨γ1, Hγ1⟩, -⟩
  iexists γ0
  iexists γ1
  iexists "string0"
  iexists "string1"
  isplit l [Hγ0]
  · iexact Hγ0
  · iexact Hγ1
```