SampCert

Foundations for Verified Differential Privacy

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Privacy is tricky

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Oversimplified privacy models

Precision leaks Mironov (2012), Tumult Labs (**2021**)

Sensitivity errors (overflow) Casacuberta et al. (**2022**)

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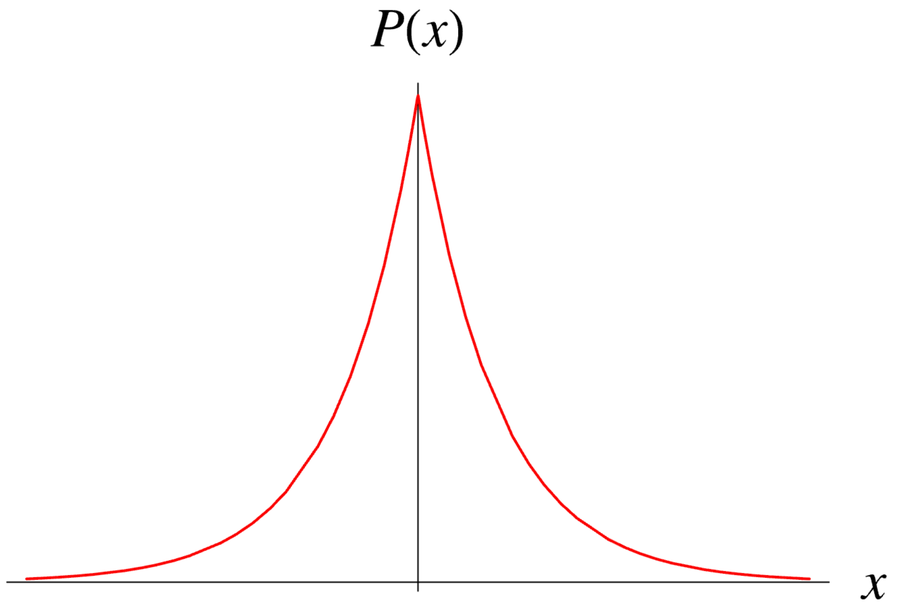
Oversimplified privacy models

On paper: $\epsilon = 10^{-6}$



mathworld.wolfram.com

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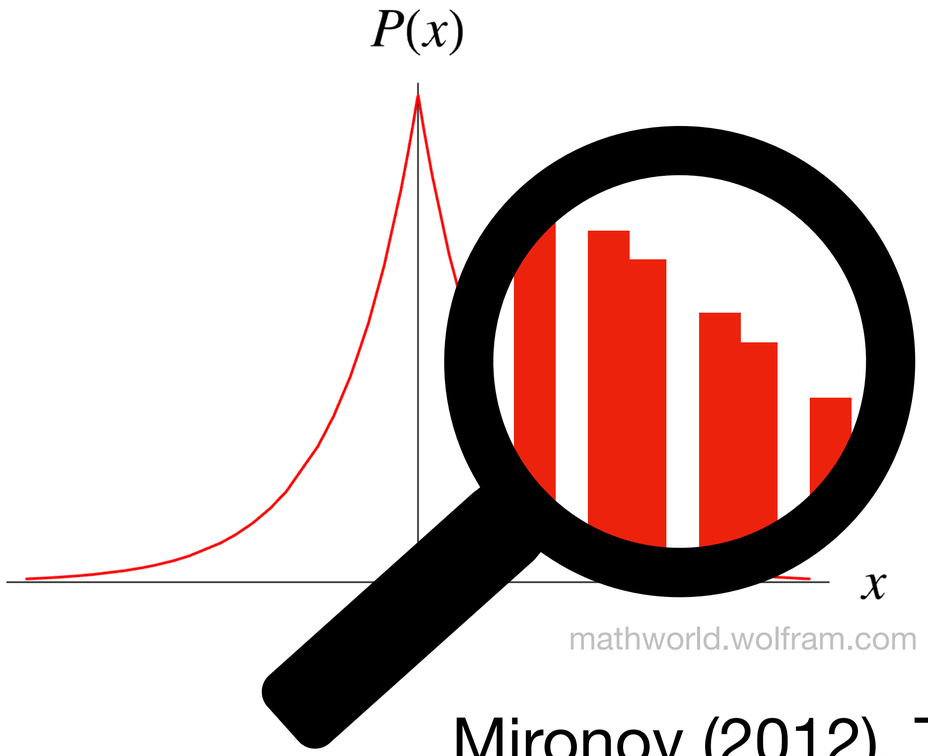

In code: 40%

Precision leaks

Sensitivity errors (overflow)

Mironov (2012), Tumult Labs (**2021**)

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Privacy is tricky

Oversimplified privacy models

Buggy implementations

Algorithmic errors Lyu et al. (2017)

Budget errors *Cebere et al. (**2026**)*

Privacy is tricky

Oversimplified privacy models

Buggy implementations

Many subtly wrong implementations of textbook SVT

Algorithmic errors Lyu et al. (2017)

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Privacy is tricky

Oversimplified privacy models

Buggy implementations

Incomplete attacker models

Timing leaks *Jin et al. (2021)*

Question:

When we say code is differentially private,

how can we check that it actually is?

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Answer:

Formal Verification

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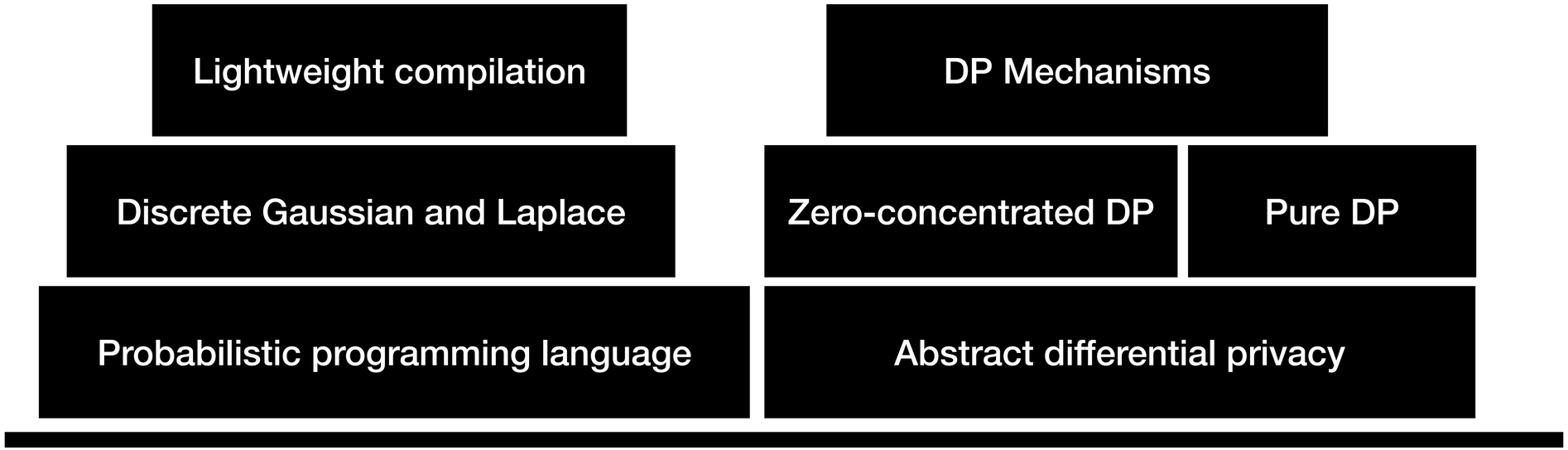
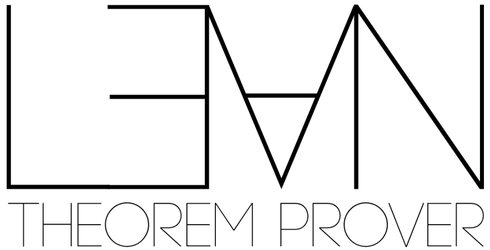
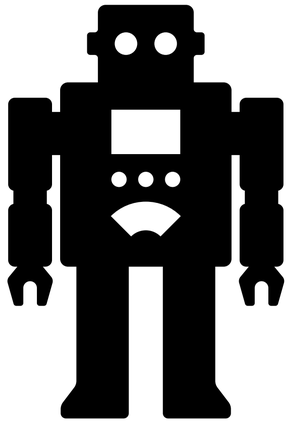
Foundations for Verified Differential Privacy





Privacy Engineer (you)



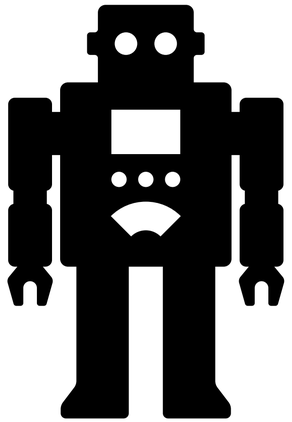
Proof Checker

Code

Sampling algorithm

Mechanism

Privacy Engineer (you)



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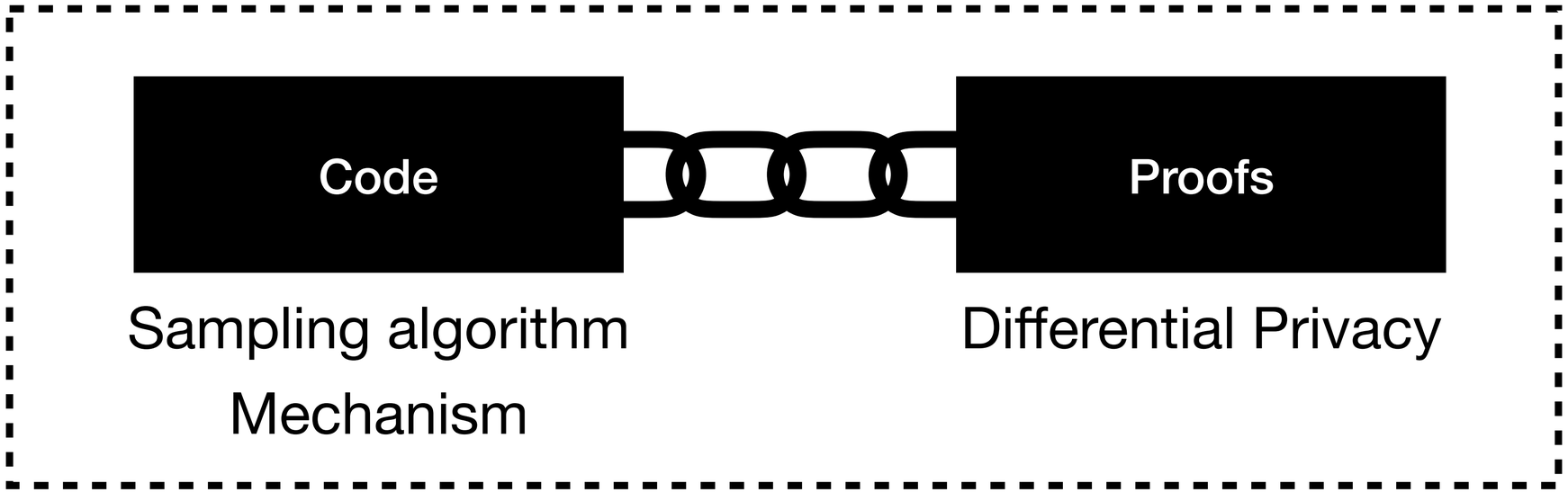

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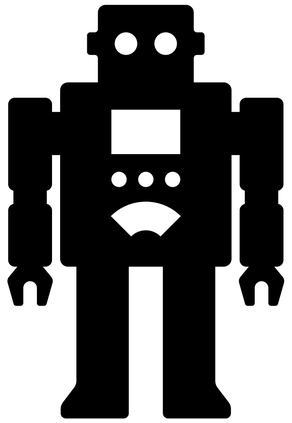

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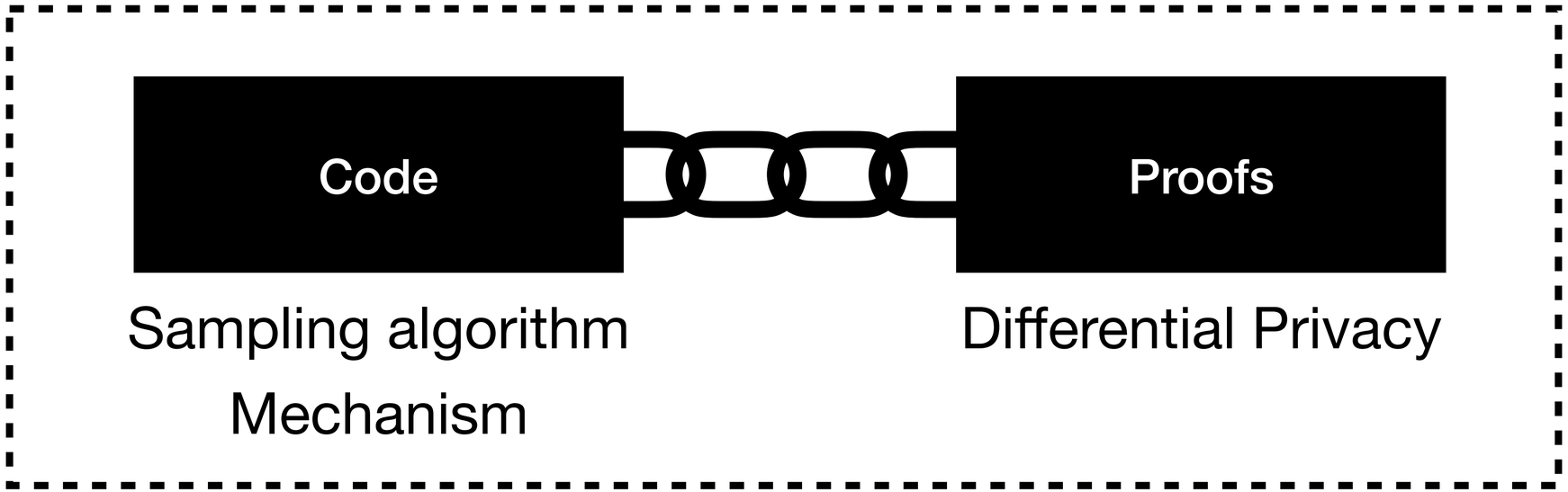

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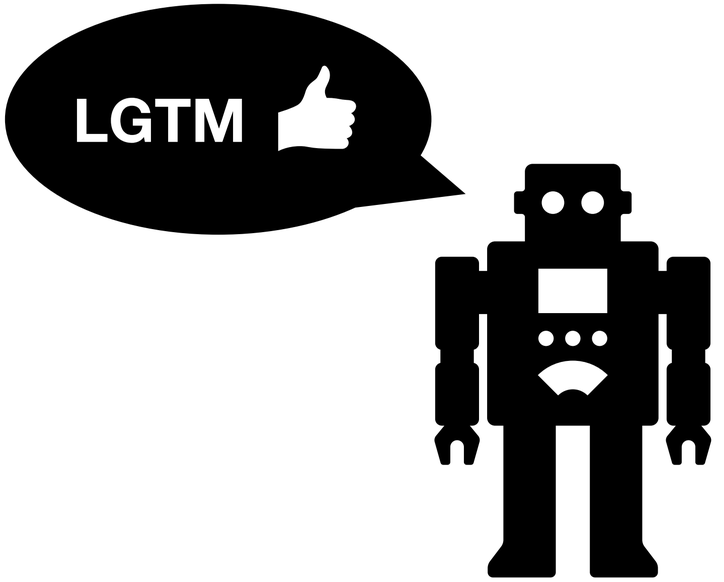
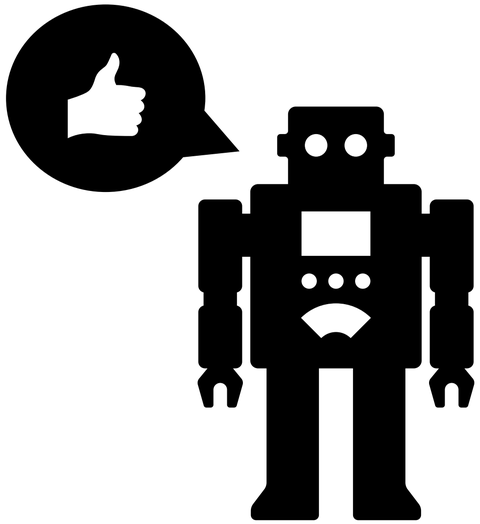
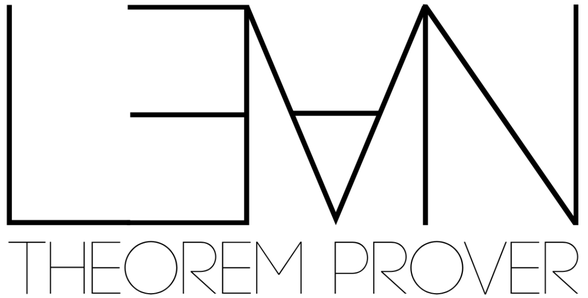


Proof Checker

What does formal methods get you?





✓ Your privacy proof is correct

Compilation of your code to binary

Coding in SampCert

Algorithm 3 Algorithm for Sampling a Discrete Gaussian

Input: Parameter $\sigma^2 > 0$.
Output: One sample from $\mathcal{N}_{\mathbb{Z}}\left(0, \sigma^2\right)$.

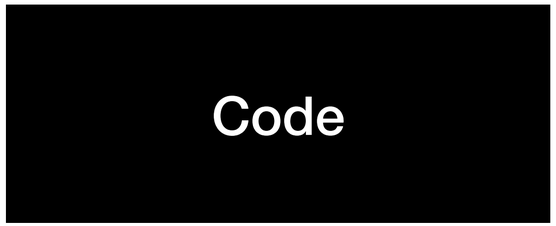
Set $t \leftarrow \lfloor \sigma \rfloor + 1$
loop
  Sample $Y \leftarrow \mathrm{Lap}_{\mathbb{Z}}(t)$
  Sample $C \leftarrow \mathsf{Bernoulli}(\exp(-(|Y| - \sigma^2/t)^2/2\sigma^2))$.
  If $C = 0$, reject and restart.
  If $C = 1$, return Y as output.

```
/-- Sample a candidate for the Discrete Gaussian with variance ``num/den``. -/
def DiscreteGaussianSampleLoop (num den t : PNat) (mix : ℕ) : SLang (Int × Bool) := do
  let Y : Int ← DiscreteLaplaceSampleMixed t 1 mix
  let y : Nat := Int.natAbs Y
  let n : Nat := (Int.natAbs (Int.sub (y * t * den) num))^2
  let d : PNat := 2 * num * t^2 * den
  let C ← BernoulliExpNegSample n d
  return (Y,C)

/-- Sample a value from the Discrete Gaussian with variance ``(num/den)``^2. -/
def DiscreteGaussianSample (num : PNat) (den : PNat) (mix : ℕ) : SLang ℤ := do
  let ti : Nat := num.val / den
  let t : PNat := ⟨ ti + 1 , by simp only [add_pos_iff, zero_lt_one, or_true] ⟩
  let num := num^2
  let den := den^2
  let r ← probUntil (DiscreteGaussianSampleLoop num den t mix) (λ x : Int × Bool ⇒ x.2)
  return r.1
```

Canone, Kamath, Steinke. *The discrete Gaussian for differential privacy.*

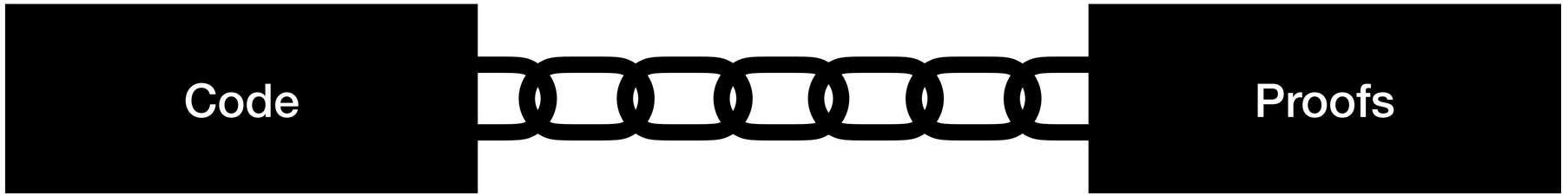
Coding in SampCert



Minimal language with:

- Pure operations
- Uniform byte samples
- Loops and recursion

Coding in SampCert



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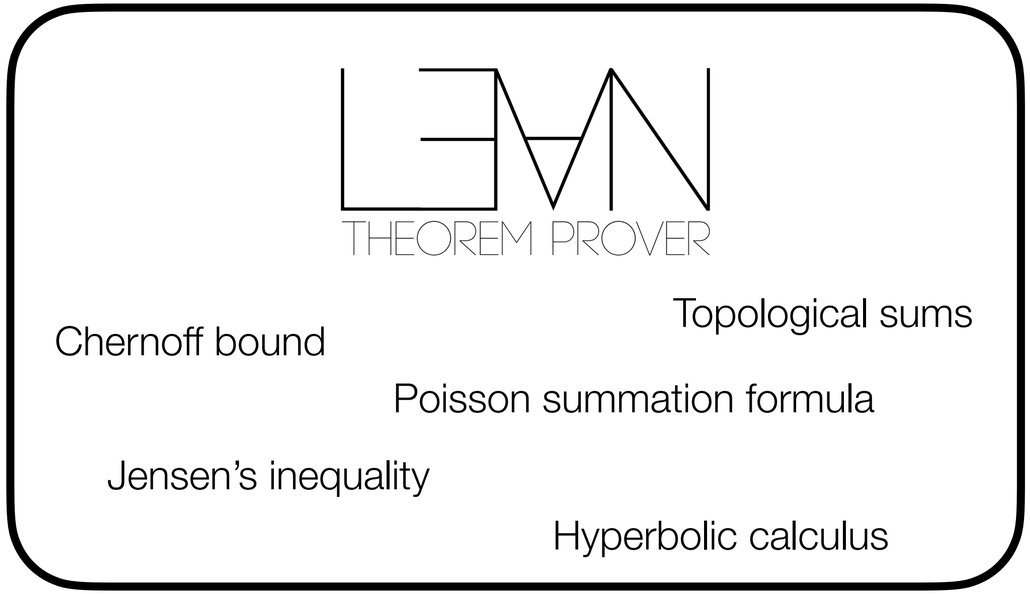
Probability Mass Functions



Proving correctness in SampCert

```
theorem DiscreteGaussianSample_apply (num den : Positive) : ∀ z : ℤ,
    DiscreteGaussianSample num den z = e ^ ((-(z - μ)^2) / (2 * σ^2)) / (Norm num den)
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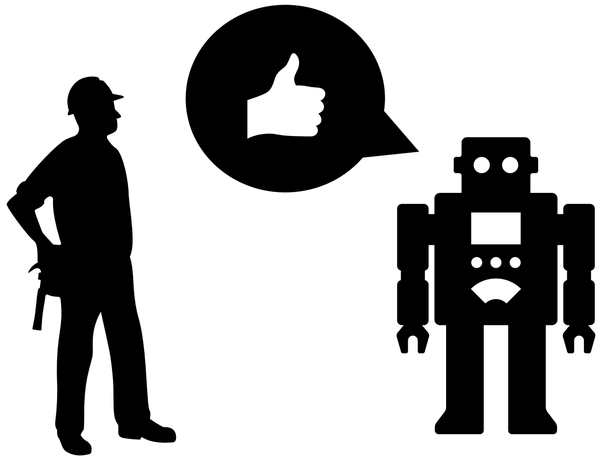
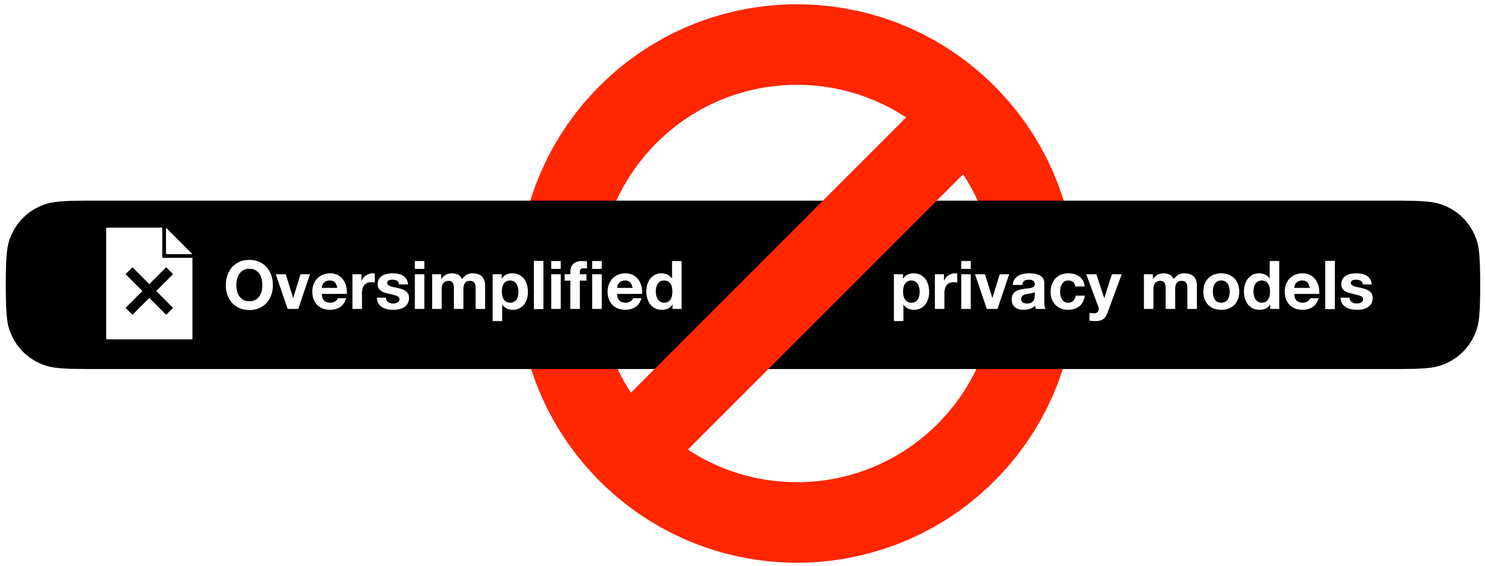
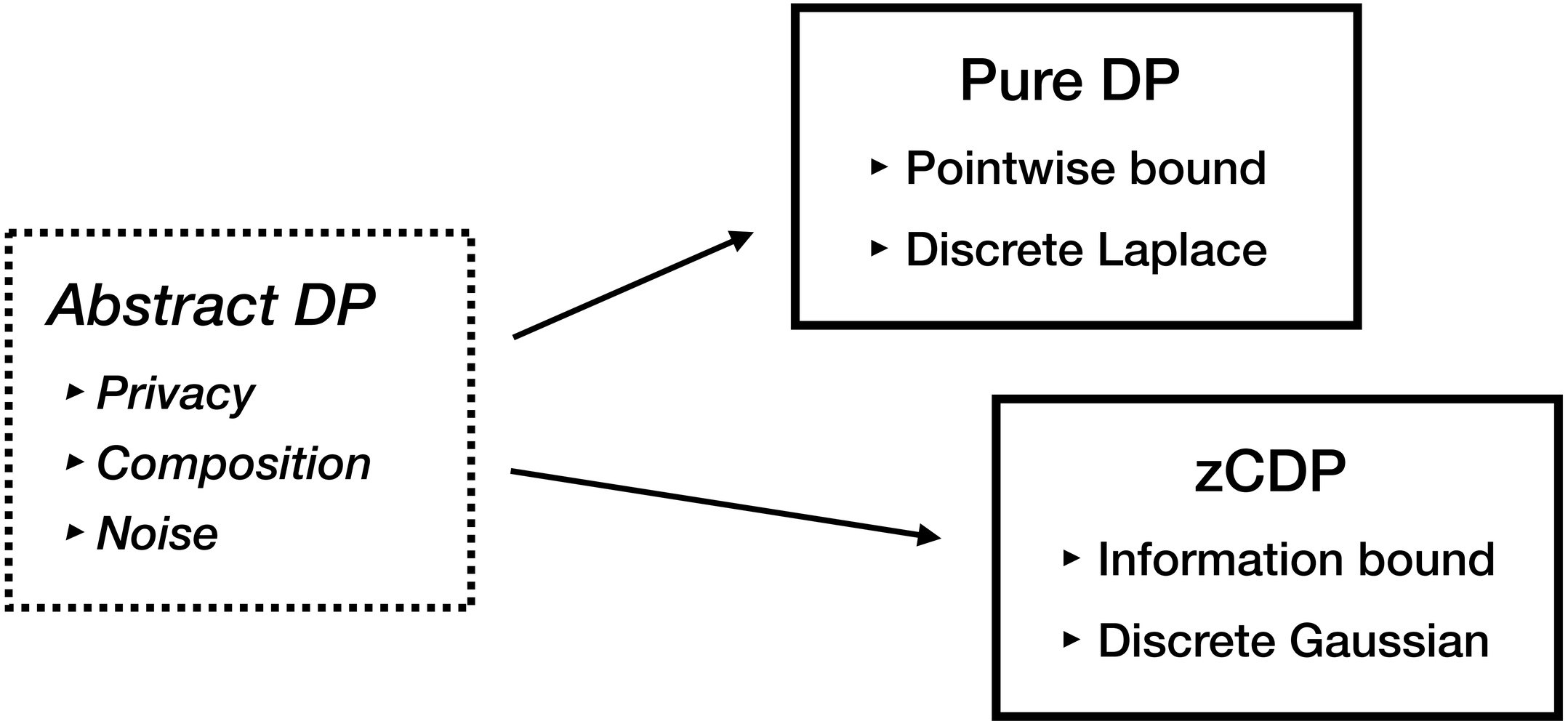


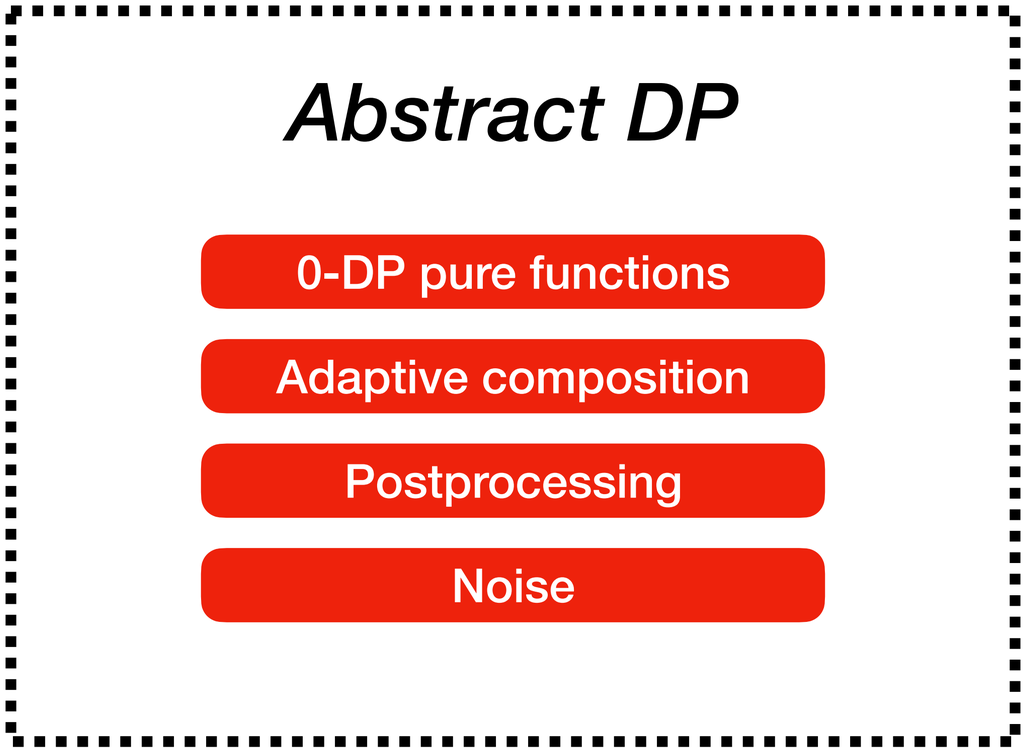


Buggy floating-point samplers fail here

Defining Privacy in SampCert

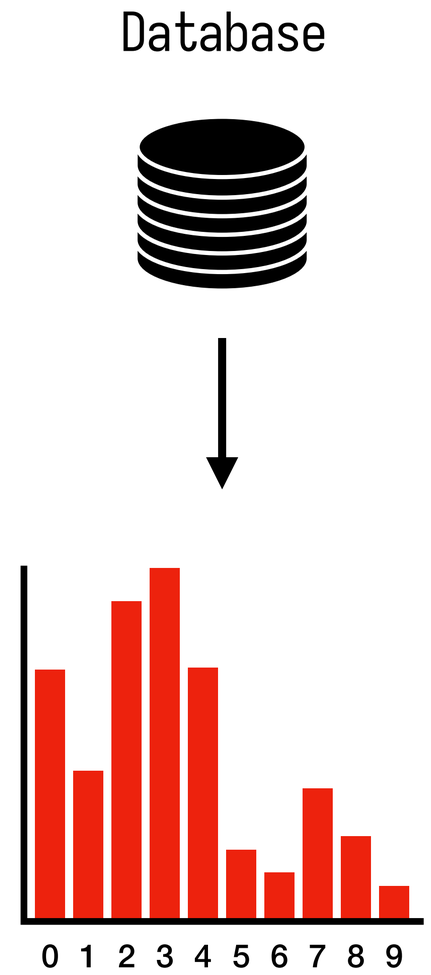


Abstract Differential Privacy



Abstract Privacy Proofs

```
def privHistogram (D : Database) (n : ℕ) := do
  match n with
  | 0 ⇒ return emptyHistogram
  | n' + 1 ⇒ do
     let h ← privHistogram D (n - 1)
     let c ← count D n + noise ε
     updateHistogram h n c
```



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    let c ← count D n + noise ε
    updateHistogram h n c
```

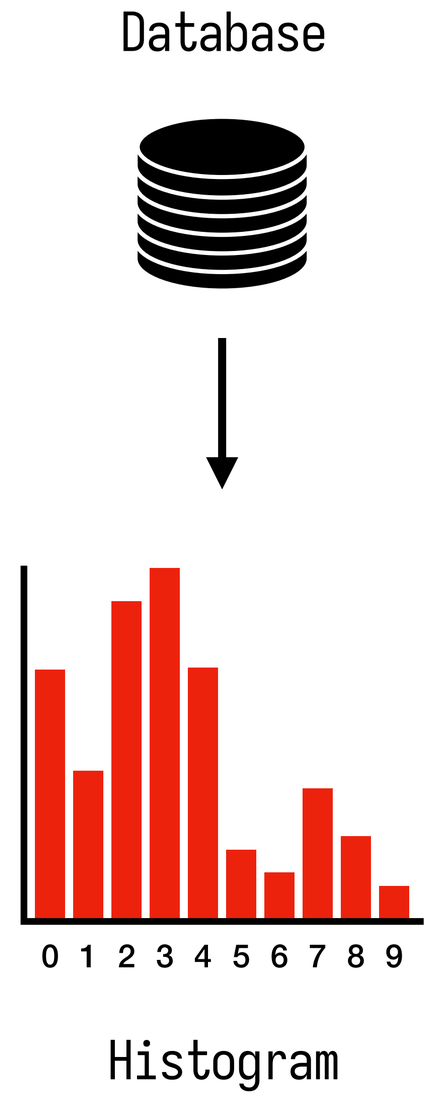
pure

composition

composition

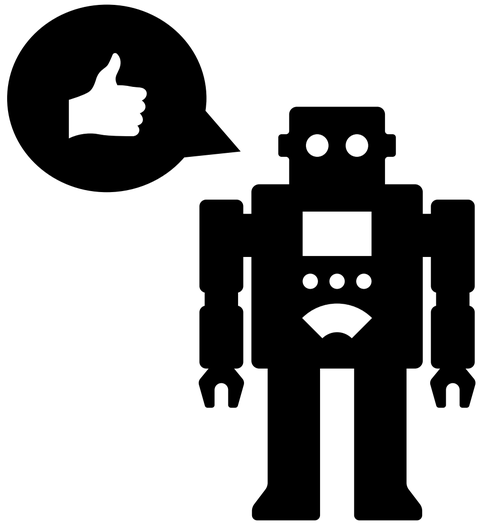
noise

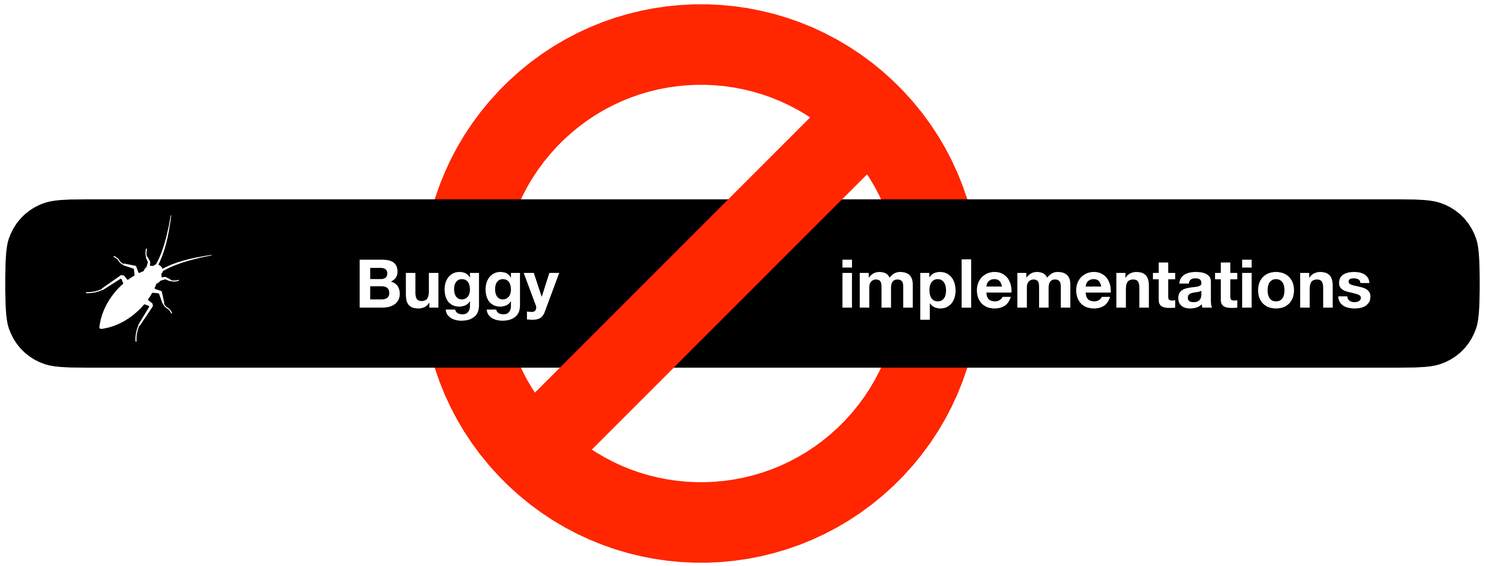
postprocessing



Advanced Privacy Proofs

- zCDP → pure DP conversion bound
- zCDP → approximate DP conversion bound
- pure DP of the sparse vector technique

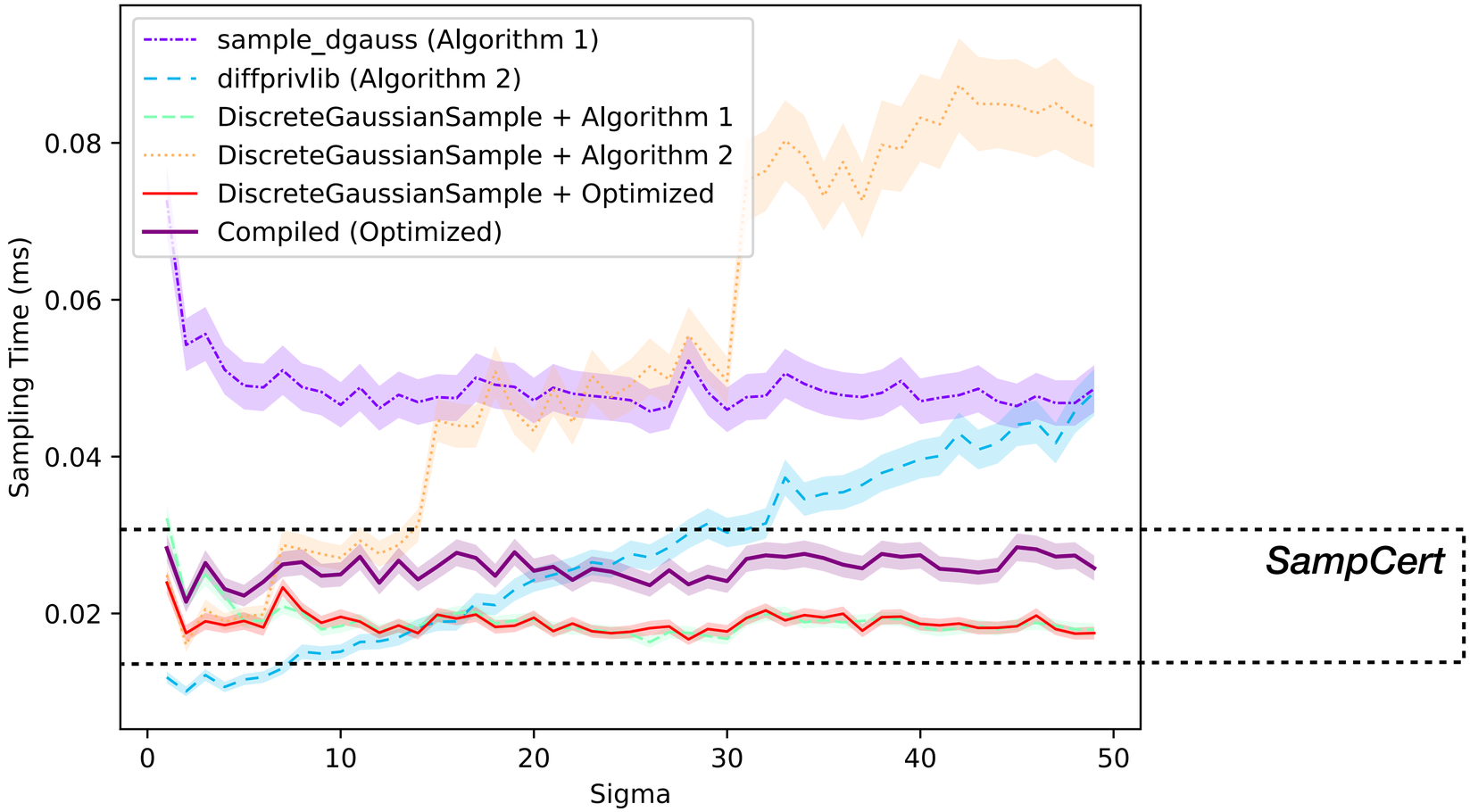




Non-private SVT implementations fail here

Executing SampCert code

Realistic performance



Lean FFI

Return

```
return v;
```

Uniform

```
unsigned char r;
read (random, &r, 1);
return r;
```

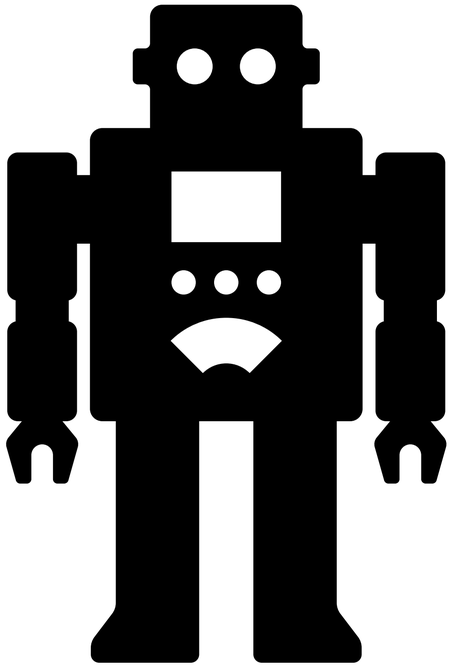
While

```
lean_object* st = init;
uint8_t c = lean_apply_1 (c, st);
while (c) {
    s = lean_apply_2 (b, st);
    c = lean_apply_1 (c, st); }
return st;
```

Bind

```
lean_object* e = lean_apply_1 f;
lean_object* p =
  lean_apply_2 (p, e);
return p;
```

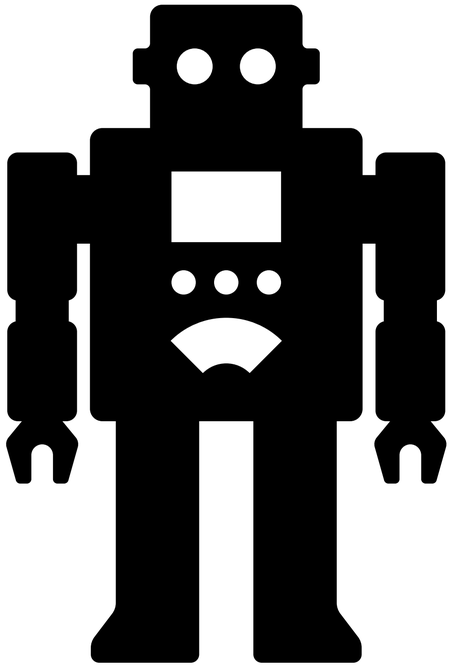
Execution is not verified



✓ High-assurance proof checker

? Conventional compiler, but not formally verified

Execution is not verified



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Ongoing work: **Incomplete attacker models**

SampCert

Foundations for Verified Differential Privacy

Paper

Verified Foundations for Differential Privacy
(PLDI 2025)

Repo

`github.com/leanprover/SampCert`

PR's Welcome!